【Grant-in-Aid for Young Scientists(S)】
Science and Engineering (Mathematical and physical sciences)

Title of Project：Strategic Research to solve certain conjectures in Arithmetic Geometry

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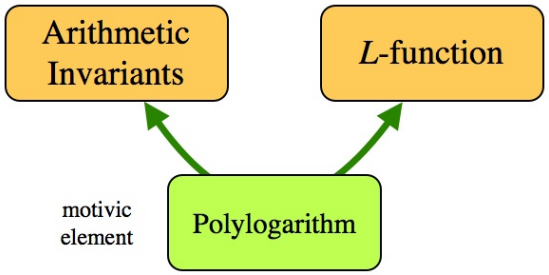
Research Area：Arithmetic Geometry, Number Theory

Keyword：Polylogarithm, theta function, *L*-function, *p*-adic *L*-function

【Purpose and Background of the Research】

The relation between arithmetically important invariants and *L*-functions has been a central theme in number theory. The class number formula giving the class number in terms of the Dedeking zeta function is a classical example. This relation has been generalized as conjectures in various direction, and was finally formulated by S. Bloch and K.Kato into the Tamagawa number conjecture (also sometimes known as the Bloch-Kato conjecture). This conjecture is a central theme in arithmetic geometry.

The difficulty of the Tamagawa number conjecture stems from the fact that one must relate arithmetic invariants and *L*-functions, which at first glance are totally different objects. The purpose of the present research is to study the "polylogarithm", a motivic object which may potentially be used to connect the two sides.



The polylogarithm has been constructed for various algebraic varieties. It was first constructed for the case of the projective line minus three points by Beilinson and Deligne. The construction has been generalized to the case of elliptic curves by Beilinson and Levin, and to the case of abelian varieties by Wildeshaus and Kings. The polylogarithm has a simple characterization, but its difficulty is in explicitly describing this object.

Recently, with T. Tsuji and S. Kobayashi, we have given a simple and explicit description of the elliptic polylogarithm in terms of the reduced theta function associated to the Poincaré bundle. In this current research, we aim to do the following:

・To study arithmetic implications of the explicit description of the elliptic polylogarithm.

・Attempt to describe the polylogarithm in other cases, eg. the case of abelian varieties.

【Research Methods】

The project will hire two to three young researchers, and the project will be conducted as a team. The main objective will be to strategically and systematically research the arithmetic implication of the explicit description of the elliptic polylogarithm.

【Expected Research Achievements and Scientific Significance】

We may be able to obtain the *p*-adic Beilinson conjecture, deeply related to the Tamagawa number conjecture, for Hecke characters of imaginary quadratic fields. We may also be able to reinterpret in terms of the elliptic polylogarithm of the result of K. Kato on the Iwasawa theory of elliptic curves.

Understanding the polylogarithm of abelian varieties seem yet to be a difficult. However, if could obtain an explicit description in an arithmetically meaningful way, then we may be able to advance our understanding of the Tamagawa number conjecture in this case.

【Publications Relevant to the Project】

・Beilinson and Levin, the elliptic polylogarithm, in: *Motives*, Proc. Symp. Pure Math. **55**, Pt. 2, pp. 123-190 (1994).
・S.Bloch and K.Kato, L-functions and Tamagawa numbers of motives, in: *the Grothendieck Festschrift*, Vol. I, pp. 333-400, Prog. Math. **86**, Birkhauser, Boston MA, 1990.
・K.Bannai, S.Kobayashi and T.Tsuji, On the de Rham and p-adic realizations of the elliptic polylogarithm for CM elliptic curves, arXiv:0711.1701v2
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【Term of Project】 FY2009-2013

【Budget Allocation】 71,800 Thousand Yen

【Homepage Address and Other Contact Information】

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