【Grant-in-Aid for Scientific Research (S)】

Graph Algorithms and Optimization: Theory and Scalable Algorithms

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Project Information	Project Number : 22H05001　　Project Period (FY) : 2022-2026 Keywords : Graph Algorithm, Graph Theory, Graph Structure

Purpose and Background of the Research

● Research background

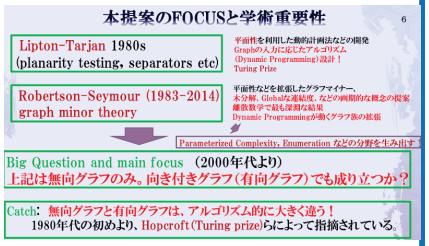
As indicated in the Master Plan 2020 of the Science Council of Japan, “the fourth science” has emerged in recent years, and information technology has become very important in almost every scientific field. The importance of the algorithmic foundation that drives this is increasing. In particular, algorithmic innovations such as current information retrieval (i.e., google) and privacy protection have led to the creation of businesses on a national scale. In the history of IT development, well-known theoretical researchers have contributed to innovative software development and various solutions at giant IT companies such as Microsoft, Google, Yahoo, AT&T, Facebook, and Amazon. For example, algorithmic innovations such as current information search technology (Google’s PageRank) and security technology (Apple’s (Local) Differential Privacy) have led to the creation of businesses on a national scale and created the current GAFA and Microsoft. What is important here is that both PageRank and Differential Privacy are basic and theoretical research on algorithms and discrete mathematics, and are not applications-oriented from the beginning. Giant IT companies such as Microsoft and GAFA have been aware of this fact since their establishment, and have provided generous support for algorithms, discrete mathematics, and combinatorial optimization. However, unfortunately in Japan, not only are there extremely few examples of theorists contributing to Japanese companies, but there are also not many examples of theorists being active. So far, only a few percent of papers by Japanese researchers have been accepted at top tier international conferences in computer science.

In order to promote innovation like the giant IT companies mentioned above, it is necessary for researchers who are well versed in mathematical theory to take on the challenge of solving social problems. The ERATO Kawarabayashi Large Graph Project (2012-2018), led by PI, aims to provide researchers who are capable of using mathematical theory. By applying it to various fields of informatics, she/he contributed to improving the status of Japan's IT basic research field. This research proposal aims to further strengthen PI's research activities to date, in particular, to disseminate research results in graph algorithms, discrete mathematics, and combinatorial optimization. We also aim to establish a global base for algorithmic research.

●Purpose of research

In the past 30 years, there has been a great deal of research using graph structures in graph for algorithm design. This trend began with algorithmic designs such as Turing Award winners, Tarjan, Hopcroft's planar graphs, and separators, and reached its peak in the 2000s. At the top of this field is ``Graph Minor Theory (GM)'' by Robertson and Seymour. They established ``Graph Minor Theory'' in a series of 23 papers with the common title ``Graph Minors (GM)'' (1986-2004). This theory has been evaluated as one of the most profound results and theory, not only in discrete mathematics but also in the field of algorithms. However, extending graph miner theory to directed graphs (graphs with oriented edges) has been unsolved for many years. In fact, it has become clear forty years ago that there is a big difference between directed and undirected graphs in terms of the amount of computation for some problems.

We already know that profound methods and results for undirected graphs cannot be easily extended to directed graphs. As a first step in extending the GM of undirected graphs to directed graphs, in 2015, together with Dr. Kreutzer, a collaborator, completely solved the "min-max conjecture of directed graph tree width and grid miner" which is conjectured by eminent researchers such as Reed, Robertson, Seymour, and Thomas in the mid-1990s (STOC'15). With this solution, we have succeeded in extending to directed graphs up to the sixth paper of Graph Miner Theory (GM). Is it possible to extend these results further to GM23? This is a central issue in current research on directed graphs. We aim to make a substantial contribution to this issue.



Our Focus of research

Expected Research Achievements

● What and how will this research attempt to clarify?

Just as Graph Miner Theory (GM) for undirected graphs required many steps (23 papers, more than 500 pages), Directed Graph Miner Theory (DGM) will also require various steps in the future, is a common understanding in the algorithm research community. The biggest interest in the algorithm research community is "how to extend the method used in GM to DGM?" As a background, regarding the graph structure, undirected graphs and directed graphs are very different. For example, in the undirected graph, Thomassen, Seymour et al. showed that the ``planar graph'' is the real obstacle for the two disjoint paths problem, and that otherwise it is tractable if we assume trivial connectivity. Thus "Good characterization" for this problem has been known for 40 years. Around the same time, however, it was shown that the corresponding problem for directed graphs is NP-hard. This fact suggests that it is difficult to extend the graph structure, which is a "good characterization" for undirected graphs, to directed graphs, and that it is necessary to develop a unique method for directed graphs. In fact, as mentioned above, PI completely solved the ``min-max conjecture of directed graph tree width and grid miner'' in 2015, but the proof is completely inadequate with the method of GM V alone, and it is unique to directed graphs We have developed many methods and tools that are completely different from the conventional ones.

In addition, PI has published the progress of DMG theory construction in the past few years (SODA'20). This work provides the algorithmic research community with a substantial contribution to the structure of directed graphs. Then, for tools of undirected graphs, we aim to break through the "difficult" situation peculiar to directed graphs by developing original tools that far surpass GM papers.

● Expected significance

Through this research, the following academic and technical contributions are expected .

1. Algorithm methods developed in individual fields of mathematics and informatics will be developed and a system will be established to provide them to each field. Specifically, the above-mentioned directed graph structure (graph miner theory), online learning, deep learning theory, etc. are targeted.
2. A joint research base will be established for algorithm science and real-world problem solving by researchers in discrete mathematics, theoretical computer science, probability theory, and combinatorial optimization.

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