【Grant-in-Aid for Scientific Research (S)】

Broad Section B

Title of Project : Representation theory of vertex algebras for the 21st century

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【Purpose and Background of the Research】

Vertex algebra is an algebraic object in mathematics, such as groups, rings and fields. Vertex algebra was introduced by Borcherds in the late 1980s as a mathematic framework for the two-dimensional conformal field theory. Vertex algebras can be also considered as a genitalization of infinite-dimensional Lie algebras such as affine Kac-Moody algebras and the Virasoro Lie algebra, since they provide basic examples of vertex algebras. Now, since they originated from two-dimensional conformal field theories, vertex algebras were initially not expected to be related with higher dimensional quantum field theories. However, various connections of vertex algebras with highest dimensional quantum field theories have been found recently. One of such connections is the 4D/2D duality, which associates a vertex algebra to any 4-dimensional N=2 superconformal field theory. It is expected that the vertex algebra gives a complete invariant of the corresponding 4D theory. Hence, it is extremely important to understand the vertex algebras appearing in the 4D/2D duality.

On the other hand, the vertex algebras appearing in the 4D/2D duality are much wilder that the vertex algebras that have been studied in the past. With Kawasetsu the PI introduced the notion of quasi-lisse vertex algebras, which provides one of the characteristics of the vertex algebras appearing in the 4D/2D duality.

An important point here is that 4D N=2 SCFTs are extremely rich theory. Hence, the corresponding vertex algebras should have extremely rich structure as well. Indeed, numerous interesting studies have been made in physics for those vertex algebras appearing in the 4D/2D duality. For instance, Beem and Rastelli conjectured that the Higgs branch of a 4D N=2 SCFT, which is a geometric object, should be isomorphic to the associated variety of the corresponding vertex algebra, which is purely an algebraic object as we have explained in the beginning. However, very little mathematical study has been made on these recent developments in physics.

In this project, we study the representation theory of quasi-lisse vertex algebras with the view point of the 4D/2D duality.

【Research Methods】

Our basis themes are

- Associated varieties of vertex algebras and Beem-Rastelli conjecture;
- Representation theory of W-algebras and Argyres-Douglas theory;
- Chiral algberas of class S and the (real) geometric Langlands program;
- Representation theory of quasi-lisse vertex algebras and Hitchin system;
- The reduction mod p method in the study of quasi-lisse vertex algebras.

We study these themes with Kawasetsu and Yamanaka who are experts in vertex algebras, with Hikita who is an expert in geometric representation theory, with Nishinaka who is an expert in the physical theory of the 4D/2D duality, with a few post-docs we are going to hire with this budget, and with many collaborators s outside Japan. We will organize international and domestic conferences, as well as schools, to communicate with collaborators and to interacts with other experts in the related topics.

【Expected Research Achievements and Scientific Significance】

This project will provide a vast generalization of the results on the representations of W-algebras obtained by the PI and his collaborators in recent years. The results are expected to impact not only on the field of representation theory, but also on the fields of geometry, integrable systems, and even of particle physics, since mathematically the 4D/2D duality can be viewed as a new framework that provides an interaction between algebra, geometry, integrable systems, and particle physics.

【Publications Relevant to the Project】

- T. Arakawa and K. Kawasetsu, Quasi-lisse vertex algebras and modular linear differential equations, In: V. G. Kac, V. L. Popov (eds.), Lie Groups, Geometry, and Representation Theory, A Tribute to the Life and Work of Bertram Kostant, Progr. Math., 326, Birkhauser, 2018.
- T. Arakawa, Chiral algebras of class S and Moore-Tachikawa symplectic varieties, arXiv:1811.01577 [math.RT].

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