【Grant-in-Aid for Scientific Research (S)】

Science and Engineering (Mathematical and Physical Sciences)

# Title of Project : Innovative Research of Geometric Topology and Singularities of Differentiable Mappings

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Research Area : Differential Topology
Keyword : Singularity, Low-dimensional Topology

## 【Purpose and Background of the Research】

Differential Topology, born in mid-20th century, gave a great shock to Mathematics community with Milnor's discovery of exotic differentiable structures, where singularities of functions played an important role. Thom proposed Catastrophe Theory based on Singularity Theory, exhibiting its applications to various fields. Later, strong analytic techniques were introduced in Topology, although they are not very constructive. With recent maturity of analytic ideas, importance of Geometric Topology, which is based on concrete constructions, is increasing. Under such circumstances, our project aims to reform Singularity Theory with concrete and constructive ideas of Geometric Topology, and to innovate formulations, concepts and techniques. We also open up a new road in Geometric Topology from singularity theoretical viewpoints, and solve important problems. We thereby establish a new research field, "Next-generation Catastrophe Theory", and give evolution to Topology.

## 【Research Methods】

We effectively use singular fibers (established by leader Saeki), singular geometric structures, mapping class groups (member Endo's contribution is essential). We use "charts" which visualize braid groups and monodromies (invented by member Kamada) in order to unify various invariants that have not been understood geometrically. Using characteristic classes and homotopy theory (members Ohmoto and Iwase's contribution is essential), we systematically study the elimination problem of singularities. We use real singularity theory techniques to attack the 4-dimensional smooth Poincaré conjecture (member Ishikawa's contribution is essential). Based on these methods, the members of the project conduct individual as well as joint researches by frequently discussing with each other. We also hold seminars and conferences in order to activate the relevant fields.

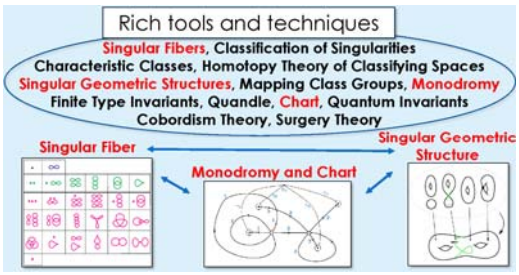


Figure 1 Research Methods

## 【Expected Research Achievements and Scientific Significance】

We get new contributions to crucial problems in Differential Topology. We open up new applications of Singularity Theory to other disciplines by using concrete and constructive ideas, and create a new research field. Developments in related fields in Mathematics are accelerated, where the versatility of Singularity Theory plays important roles. By activating Catastrophe Theory, we aim to solve problems in industry, propose new methods in other disciplines in science, and we thereby spread our results to other fields.


Figure 2 Research Outcomes

## 【Publications Relevant to the Project】


- R.A. dos Santos, M.A.B. Hohlenwerger, O. Saeki and T.O. Souza, New examples of Neuwirth-Stallings pairs and non-trivial real Milnor fibrations, Ann. Inst. Fourier (Grenoble) 66 (2016), 83-104.
- Saeki, Topology of singular fibers of differentiable maps, Lecture Notes in Math., Vol.1854, Springer-Verlag, 2004.


【Term of Project】 FY2017-2021

【Budget Allocation】 62,800 Thousand Yen

## 【Homepage Address and Other Contact Information】

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