【Grant-in-Aid for Scientific Research (S)】

Science and Engineering (Mathematical and Physical Sciences)

Title of Project : Fusion of Birational Geometry and Theory of Periods; A New Era for Studies of Mirror Symmetry

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Research Area : Mathematics, Geometry
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【Purpose and Background of the Research】

A geometric object has quantities reflecting its characteristics. For a rectangle, (height) × (width) and (height) ÷ (width) represent its size and shape. For another rectangle whose width is the inverse of the original one, its size and shape become the original shape and size. One has a pair of rectangles replacing the role of the size and shape. Mirror symmetry is, such as in this example, a symmetry exchanging two kinds of geometric features.

Mirror symmetry is an equivalence between two topological string theories called A- and B-models, associated to symplectic geometry and algebraic geometry, which yields qualitative and quantitative conjectures, relates wide range of mathematics and deepens classical one along with new findings. It is important to elucidate the mathematical truth behind, with ``physical ideas, objects and methods".

There are two particularly important issues. One is to prove the homological mirror conjecture, an equivalence between two categories associated to A- and B-models. The other is a derivation from the homological mirror conjecture of the classical one, an equivalence between Gromov-Witten theory and the deformation theory. It is to develop the theory of B-periods for categories, including primitive forms and flat (Frobenius) structures. Towards these issues, many important results have been obtained.

On the other hand, for a further understanding of birational geometry, especially, the minimal model theory, studies by categorical and mixed-Hodge theoretical methods have been extensively done. Quite recently, there appeared a new approach to study derived categories, the categorical dynamics, which also shows the importance of such methods.

By fusion of birational geometry and theory of periods, we evolve both of them drastically, deepen further understanding of mirror symmetry and contribute to traditional important problems.

【Research Methods】

Settlement of various problems in the following:

1. Fundamental studies of nc-Hodge structures and the categorical dynamics. Basic research toward the theory of quantum primitive forms.
2. Studies of (nc-) birational geometry, especially the minimal model theory, by the categorical and the (nc-) Hodge-theoretical methods.
3. Further understanding of mirror symmetry and period mappings relating GW-invariants, primitive forms and Weyl group invariants.

The plan will be carried out in the following way:

① Individual and joint research by our research system, consisting of research team members, cooperation researchers and collaborators.
② The activation and the further development by the employment of post-doctoral fellows.
③ Research origination and exchanges by our seminars, workshops and annual conferences.

【Expected Research Achievements and Scientific Significance】

Not only large contribution in the state-of-the-art studies of mirror symmetry, our research will lead to a lot of new knowledge in traditional problems with a history of more than 100 years. Provision of invariants and methods to birational geometry and elucidation of the interactions among singularities, orbifolds and root systems, are expected.

【Publications Relevant to the Project】

・ Wolfgang Ebeling, Sabir M. Gusein-Zade, Atsushi Takahashi, Orbifold E-function of Dual Invertible Polynomials, Journal of Geometry and Physics 106 (2016), 184 – 191.
・ Yuuki Shiraishi, Atsushi Takahashi, On the Frobenius Manifolds for Cusp Singularities, Advances in Mathematics 273 (2015), 485 – 522.

【Term of Project】 FY2016-2020

【Budget Allocation】 79,900 Thousand Yen

【Homepage Address and Other Contact Information】

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