【Grant-in-Aid for Scientific Research (S)】

Science and Engineering (Mathematical and Physical Sciences)

Title of Project : Multiple Zeta Values and Functions

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Research Project Number : 16H06336 Researcher Number : 70202017
Research Area : Algebra
Keyword : Number Theory, Arithmetic Geometry, Low-dimensional Topology

【Purpose and Background of the Research】

Multiple zeta values and multiple zeta functions are defined by the following nested infinite series.

$$\zeta(k_1,\dots,k_r) := \sum_{0<m_1<\cdots<m_r} \frac{1}{m_1^{k_1}\cdots m_r^{k_r}}$$

When the arguments are all positive integers, this is called the multiple zeta value, whereas as a function of complex variables, this is called the multiple zeta function. When the number of variables is one, this is nothing but the celebrated Riemann zeta function. We call here in general "multiple zetas" for those values and functions including more generalized versions. It was Euler who initiated the study of these objects, but it is since about two decades that active research has been explored in connection to various areas in mathematics as well as mathematical physics.
During the period of our research project, we try to reveal connections between various results and conjectures on multiple zeta values that have been made since two decades, and try to find a unified point of view, which may probably be still hidden behind. Also, we develop analytic and p-adic theory of multiple zeta functions, and together with the theory of multiple zeta values, we will be trying to contribute new development in the area of multiple zetas.

【Research Methods】

Main topics of research will be:
--- Relations and algebraic structures of multiple zeta values,
--- Analytic and p-adic theories of multiple zeta functions,
--- Finite multiple zeta values, p-adic multiple zeta values, and motivic multiple zeta values,
--- Multiple polylogarithms,
--- Multiple zeta values and quantum invariants from the viewpoint of arithmetic topology,
--- Poly-Bernoulli numbers,
--- Understandings from the Galois side and the combinatorics side.

Members of the project will conduct independent as well as joint works on some of these topics, communicating from time to time with each other on their progress.
We will occasionally have seminars, workshops, and conferences in order to meet with each other and discuss our research progress.

【Expected Research Achievements and Scientific Significance】

Although the progress in the area since these two decades is enormous, there still remain many unsolved problems and conjectures in the field of multiple zetas like the Broadhurst-Kreimer conjecture. Certainly we will have a better understanding toward such conjectures and of mutual connections among those coming from different backgrounds.
Recent works on finite multiple zeta values, desingularized multiple zeta functions, multiple zeta functions associated to root systems, are promising examples of fruitful establishments. Applications to other branches, such as an application of poly-Bernoulli numbers to combinatorics, will also be expected.

【Publications Relevant to the Project】

・M. Kaneko, K. Ihara and D. Zagier, Derivation and double shuffle relations for multiple zeta values, Compositio Math. vol. 142-02, pp 307--338, (2006).
・M. Kaneko, K. Tasaka, Double zeta values, double Eisenstein series, and modular forms of level 2, Math. Ann. vol. 367, pp 1091-1118, (2013).

【Term of Project】

FY2016-2020

【Budget Allocation】

75,400 Thousand Yen

【Homepage Address and Other Contact Information】

http://www2.math.kyushu-u.ac.jp/~mkaneko/kibans/en/index.html