

【Grant-in-Aid for Scientific Research (S)】

Science and Engineering (Mathematical and Physical Sciences)



Title of Project : The Birth of Modern Trends on Commutative Algebra and Convex Polytopes with Statistical and Computational Strategies

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Research Area : Mathematics

Keyword : Gröbner basis, binomial ideal, convex polytope

【Purpose and Background of the Research】

Following the success of JST CREST mathematics project “Harmony of Gröbner Bases and the Modern Industrial Society,” known as Hibi project, the present research continues the bringing up of young mathematicians and develops original ideals on pure mathematics arising from Hibi project with maintaining its steady research organization of algebra, statistics and computation.

First, with considering the historical background that commutative algebra on monomial ideals was born about 40 years ago in connection with combinatorics, commutative algebra on binomial ideals is created in connection with statistical models and designs of experiments.

Second, from the viewpoint of computation of Pfaffian systems of 0-dimensional ideals, the trio of convex polytopes, A -hypergeometric systems and statistical distributions is studied. In particular, the study on A -hypergeometric systems associated with convex polytopes yields a new and wide class of innovative models of statistical distributions.

【Research Methods】

(Strategy A) The theory of determinantal ideals has developed in connection with invariant theory, representation theory and combinatorics. Recently, under the influence of algebraic statistics on contingency tables, the algebraic study on binomial ideals generated by 2-minors becomes fashionable. One of the problems is to determine when a binomial ideal generated by 2-minors is prime and possesses a quadratic Gröbner basis.

(Strategy B) Based on the fact that Markov bases of designs of experiments correspond to systems of generators of toric ideals associated with designs, the algebraic theory of toric ideals of two-level fractional factorial designs is established and, furthermore, the possibility of computation of Markov bases of multi-level designs is researched.

(Strategy C) Order polytopes, which define toric ideals of distributive lattices, give a nice class of A -hypergeometric systems whose Pfaffian systems have the growth rate of the polynomial order. This observation naturally yields the temptation of discovering distinguished classes of convex polytopes with the particular property and of

making the database of innovative A -distributions. (Strategy D) The algebraic theory of nested configurations was founded by the feedback from statistics. Convex polytopes and toric ideals arising from nested configurations as well as centrally symmetric configurations are studied and their statistical significance is discussed.

【Expected Research Achievements and Scientific Significance】

This research can rapidly develop commutative algebra on binomial ideals and create a new world of commutative algebra with emphasis on the contribution to algebraic statistics. Moreover, the traditional theory of polytopes is dramatically reorganized and a new trend on algebraic combinatorics on convex polytopes is tempted.

【Publications Relevant to the Project】

- T. Hibi, Ed., “Gröbner Bases: Statistics and Software Systems,” Springer, 2013.
- S. Aoki, T. Hibi and H. Ohsugi, Markov chain Monte Carlo methods for regular two-level fractional factorial designs and cut ideals, *J. Statist. Plann. Infer.* **143** (2013), 1791--1806.
- V. Ene, J. Herzog, T. Hibi and F. Mohammadi, Determinantal facet ideals, *Michigan Math. J.* **62** (2013), 39--57.
- J. Herzog and T. Hibi, “Monomial Ideals,” GTM 260, Springer, 2011.
- S. Aoki, T. Hibi, H. Ohsugi and A. Takemura, Gröbner bases of nested configurations, *J. Algebra* **320** (2008), 2583--2593.

【Term of Project】 FY2014-2018

【Budget Allocation】 137,700 Thousand Yen

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