【Grant-in-Aid for Scientific Research(S)】
Science and Engineering (Mathematical and physical sciences)

Title of Project : Elucidations on uninvestigated problems related to the criticality of nonlinear dissipative and dispersive structures in mathematical models

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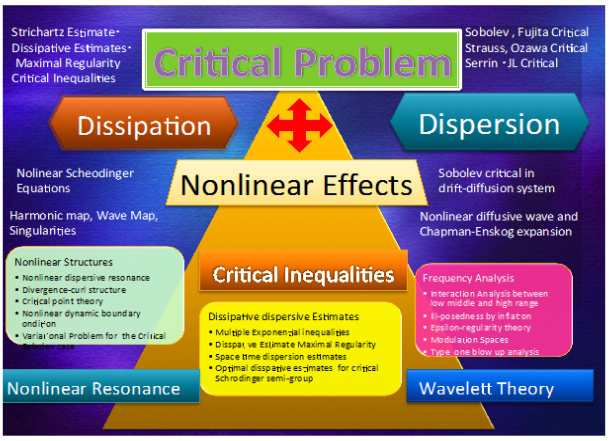
Research Area : Real analysis, Harmonic analysis, Partial differential equations.
Keyword : Nonlinear partial differential equations.

【Purpose and Background of the Research】

Many mathematical models are described by nonlinear partial differential equations and such equations typically have both linear and nonlinear structures therein. The linear part is described by partial differential operators by local space-time variables and the nonlinear part is produced by the interaction between different physical quantities and the linear structure stabilize the system while the nonlinear part causes instability of the model. Between those effects, there exists a sort of problems where the both effects are analytically balanced. We call this type of problem as the "Critical Problems" and it is our main subject of this project. Problems of this type are interesting both from an applied and a theoretical mathematical point of view. They often lead to new and fascinating open problems.

A serious difficulty in the study of such "critical problems", is that the analysis derived through perturbation theory is not directly applicable and a new methodology has to be developed.



【Research Methods】

The critical problems are in general equipped by standard and natural structures corresponding to conservation laws, for the conservation of mass, momentum and energy. These standard structures are given by entropy dissipation, Galilei invariance as well as by conformal invariance. These structures are essential in the study of the critical problems. One of the main tools used for this research are variational methods. Also fundamental for our study is the possibility of developing new functional inequalities of critical type. In particular with the aid of functional and harmonic analysis tools such as real interpolation methods, we develop a new type of inequalities which will be then used to better understand the ``critical problems" and even beyond the critical problem.

【Expected Research Achievements and Scientific Significance】

Important unsolved problems, such as the well known millennium ones, which describe central issues in mathematics are still wide open. An important challenge is to find unified methods to overcome the difficulties that stem from the interaction of both dissipative and dispersive behaviors. We believe that a finer analysis on the nonlinear structure make us possible to treat those unsolved problems after establishing a new type of critical estimates for linear and nonlinear structures including dissipative and dispersive estimate such as Strichartz estimate and maximal regularity estimates.

【Publications Relevant to the Project】

- 「Nonlinear Partial Differential Equations in Future」 Eds. H. Kozono, T. Ogawa, M. Misawa, Nihon-Hyoron-sha (Japanese) 300pp, 2007
- 「Real Analytic Method for Nonlinear Evlusional Partial Differential Equations」 Springer-Verlag Japan, (Japanese) 430pp, 2013

【Term of Project】 FY 2013-2017

【Budget Allocation】 132, 700 Thousand Yen

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