【Grant-in-Aid for Scientific Research(S)】
Science and Engineering (Mathematical and physical sciences)

Title of Project : Moduli spaces of algebraic varieties and self-morphisms

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Research Area : Mathematics
Keyword : algebraic geometry, complex geometry, representation theory, complex analysis, arithmetic geometry

【Purpose and Background of the Research】

While algebraic geometry has its own methods and problems, it has remarkable application to other fields also. Application has been made often via moduli spaces but application via self-morphism is increasing recently. Neighboring fields of complex dynamics are yielding interesting results, on such as entropy of self-morphisms and Cremona transformations, which feed back to algebraic geometry. In this project, we aim further development on problems on algebraic varieties and moduli, by adding viewpoint of self-morphism and by learning problems and research methods from neighboring fields.

【Research Methods】

Project members form several research teams, according to their research themes, to collaborate, and aim at better results and synergies

1. The first team will study moduli spaces and their compactification, centering round Enriques surfaces and Calabi-Yau varieties.

Mukai already refines the notion of root systems which Nikulin defined for Enriques surfaces. He applies it for many problems on Enriques surfaces. In neighbor of his study, appears interesting candidates of infinite automorphism groups and their higher-dimensional analogue. He is trying to find an interesting automorphism of infinite order among them.

Another study of Enriques surfaces are made via Borcheds automorphic forms. Generalizing an explicit computation of its value, one may expect deeper understanding on the period map.

2. A self-morphism of a space can be considered as a discrete dynamical system. Although deterministic, chaotic phenomena is known for such systems. Available methods are very restrictive for such systems. But for self-morphism of algebraic varieties, many properties are being clarified by algebraic, complex analytic and potential theoretic methods. The second team will study dynamical theoretic property from these viewpoint.

3. The cluster algebra and its quantization, introduced by Fomin-Zelevinsky, are found to have connections with many research fields and studied actively in recent years. On one hand, stimulated by the work of Hernandez-Leclerc, Nakajima has succeeded to realize the cluster algebra in the Grothendieck ring of category of representation of convolution algebra consisting of perverse sheaves on a graded quiver variety. On the other hand, a relation is known between cluster algebra and a wall-crossing formula of generalized invariants of Donaldson-Thomas type in Calabi-Yau categories. These are similar but a precise similarity is yet unclear. The third team will clarify this point and understand cluster algebra more deeply.

【Expected Research Achievements and Scientific Significance】

A recent study suggests a possible construction of many Enriques surfaces with explicit defining equations and with explicit infinite groups of automorphisms, and also of a self-map of moduli spaces of Enriques surfaces of certain type. Together with a close connection with Borcherds automorphic forms, Enriques surface looks the most suitable subject for which one develop a 2-dimensional analogue of the theory of elliptic curves. One may expect a generalization of classical results to Enriques surface.

【Publications Relevant to the Project】

- Shigeru Mukai: Kummer's quartics and numerically reflective involutions of Enriques surfaces, J. Math. Soc. Japan, 64, 231-246 (2012).
- Hiraku Nakajima: Quiver varieties and cluster algebras, Kyoto J. Math., 51, 71-116 (2011).

【Term of Project】 FY2013-2017

【Budget Allocation】 42, 800 Thousand Yen

【Homepage Address and Other Contact Information】

http://www.kurims.kyoto-u.ac.jp/~mukai/index-j.html