【Grant-in-Aid for Scientific Research(S)】
Science and Engineering (Mathematical and physical sciences)

Title of Project : Developments in Interactions between Algebraic Geometry and Integrable Systems

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Research Area : Mathematics
Keyword : Algebraic Geometry, Integrable system, Differential Geometry, Complex manifolds

【Purpose and Background of the Research】

Inaba, Iwasaki and Saito constructed moduli spaces of stable parabolic connections with regular singularities, and showed that Riemann-Hilbert correspondences are proper, surjective bimeromorphic morphisms. More recently, Inaba and Saito extended this result to the case of irregular singularities under mild conditions. Establishing the geometry of Riemann-Hilbert correspondences, we gave a rigorous proof of the geometric Painlevé property of nonlinear differential equations arising from isomonodromic deformations of linear connections. In addition, there have been remarkable progress in the birational geometry and various quantum invariants in recent years, and one may expect further developments in interactions between algebraic geometry and integrable systems. From these backgrounds, our research objectives can be listed below.

1. Study of moduli spaces of stable parabolic connections with irregular singularities and the geometry of Riemann-Hilbert correspondences.
2. Study of the minimal model theory in the birational geometry and its applications to phase spaces and geometric Langlands conjecture.
3. Study of quantum invariants and their correlation functions and mathematical understanding of mirror symmetry.

【Research Methods】

Project members in Kobe will work together with other related researchers from the various fields like differential equations, integrable systems, birational geometry, moduli theory, mirror symmetry, representation theory, derived category and mathematical physics. According to the research purposes, we encourage each project member to promote each individual research and collaboration with other members. In order to share newly obtained results and new problems to be solved, we will organize workshops and research conferences. Keeping in touch with related overseas researchers, we will update the research project and obtain fruitful collaboration with them. Maintaining the home page of the project, we deliver the research information. Employing young PD researchers, we will promote the researches in next generations.

【Expected Research Achievements and Scientific Significance】

We can understand the geometric structures of isomonodromic differential equations by algebraic construction of the phase space. We will also have more deep understanding of the moduli space of representations of the fundamental group of a punctured curve, analytically isomorphic to the moduli of connections via Riemann-Hilbert correspondence. Furthermore we expect to obtain more deep geometric structures of the moduli space of connections like symplectic structures, canonical coordinates by apparent singularities, Lagrangian subvarieties, Laplace transforms, which may give us mathematical foundation of understanding the geometric Langlands correspondence and the mirror symmetry.

【Publications Relevant to the Project】

[1] F. Loray, M.-H. Saito, C. Simpson, Foliations on the moduli space of rank two connections on the projective line minus four points, Sém. et Cong. 27, (2012), 115-168
[2] M. Inaba, K. Iwasaki and M.-H. Saito, Moduli of Stable Parabolic Connections, Riemann-Hilbert correspondence and Geometry of Painlevé equation of type VI, Part I, Publ. Res. Inst. Math. Sci., 42, (4), (2006), 987--1089.
[3] T. Mochizuki, Asymptotic behaviour of tame harmonic bundles and an application to pure twistor D-modules. I, II , Mem. Amer. Math. Soc., 185, (2007), no. 869-870

【Term of Project】 FY2012-2016
【Budget Allocation】 94,900 Thousand Yen
【Homepage Address and Other Contact Information】
http://www2.kobe-u.ac.jp/~mhsaito/ftop.html