【Grant-in-Aid for Scientific Research(S)】
Science and Engineering (Mathematical and physical sciences)

Title of Project : Concentration Phenomena and Structure of Solution for Nonlinear Evolution Equations

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Research Area : Mathematical and Physical Sciences, Mathematics, Analysis
Keyword : Functional Equations

【Purpose and Background of the Research】

One of the most important problems in nonlinear partial differential equations is to characterize “concentration phenomena”, which are a kind of singularity in a wide sense. This wide-sense singularity does not mean the irregular part of solution unlike the usual usage of the word “singularity”. This implies some properties of solutions, which may prevent a possible property we naturally expect of a solution. For example, when we consider the nonlinear scattering problem for nonlinear Klein-Gordon or Schödinger equations with power nonlinearity in the energy space, there is a possibility that a solution may be localized in some region for a long time like a solitary wave, because those equations are invariant under space-time translation and reflection. If one can exclude this possibility of localization, one can construct the nonlinear scattering theory. This kind of wide-sense singularity could happen if some quantity of solution, for instance, the p-integral norm of solution would concentrate on some region in the position space or momentum space. The aim of our research is to analyze the mechanism of formation of wide-sense singularity such as concentration phenomena by using harmonic analysis techniques and numerical simulations for nonlinear dispersive or wave equations, reaction diffusion equations and incompressible Navier-Stokes equations.

【Research Methods】

From a theoretical point of view, we analyze concentration phenomena by using the Fourier restriction norm method, the I-method and the minmum blowup solution argument and we try to improve these methods. From a numerical point of view, we pursue our study about how rigorously we can reproduce the singularity by using the verified numerical computation. For that purpose, we need to establish the theory of verified numerical computation which can cover functions taking infinity as values. Since mathematical theory handles abstract objects, it is important that we discuss problems with each other face to face. So, we make every effort to invite and send researchers to Kyoto University and other institutes as well as to organize scientific meetings. Furthermore, we invite young researchers to join us in this project to hire postdocs.

【Expected Research Achievements and Scientific Significance】

The Fourier restriction norm method is helpful for the close investigation of time local estimates and the I-method has been developed to the time global estimates of weak solutions. The minimal blowup solution argument recently exploited is useful for the classification of global behavior of general solutions. We try to combine these three methods to analyze the concentration phenomenon. As a result, we expect to have an improvement of these methods, which would be very helpful for the global existence theory and the nonlinear scattering theory of weak solution. Moreover, it is also important how one can handle the concentration phenomenon numerically. In this direction, we may expect to develop the scheme which can cover the infinity. In this respect, one may expect that our prospective results could be applied to problems in other fields.

【Publications Relevant to the Project】

・K. Nakanishi, H. Takaoka and Y. Tsutsumi, Locla well-posedness in low regularity of the mKdV equation with periodic boundary condition, Discrete Contin. Dyn. Syst., 28(2010), 1635-1654.
・J. Ginibre, Y. Tsutsumi and G. Velo, On the Cauchy problem for the Zakharov system, J. Funct. Anal., 151(1997), 384-436.

【Term of Project】 FY2011-2015

【Budget Allocation】 57,700 Thousand Yen