【Grant-in-Aid for Scientific Research(S)】
Science and Engineering (Mathematical and physical sciences)

Title of Project : Development of Analysis on Evolving Pattern for Complicated Phenomena

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Research Area : Mathematical and physical sciences / Mathematics / Global analysis (4105)
Keyword : Nonlinear Phenomena

【Purpose and Background of the Research】

It is important to analyze evolving pattern and structure in a precise way not only in mathematics but also in understanding complicated phenomena in various science and technology.

Analysis has studied various evolving quantity based on concept of limit and convergence while one of the purpose of geometry is a classification of shapes; see the Encyclopedic Dictionary of Mathematics, 4th edition, 2007 (in Japanese). For last 20 years based on development of nonlinear analysis we are gradually able to treat evolving pattern. To some extent evolving nonsmooth shapes can be tracked in a mathematically rigorous way. For example, it is our typical successful example that an evolving surface can be tracked after it pinches and developed singularities when it evolves by the mean curvature flow equation (describing motion of grain boundaries of metals) by establishing a level set method based on the theory of viscosity solutions.

Our goal is to clarify complicated behaviors and properties of nonsmooth solutions of important nonlinear partial differential equations (PDEs) describing evolving shape and pattern. We develop the theory of viscosity solutions, variational analysis, asymptotic analysis, real analysis, stochastic analysis. Based on these developments we intend to form a new field 'Analysis on Evolving Pattern' which focuses study of evolving pattern by mathematical analysis in a systematic way. Moreover, we intend to apply our results to related applied fields like the theory of crystal growth, image processing. We also explore applications to new fields which is seemingly less related by interpreting complicated phenomena by some evolving pattern.

【Research Methods】

Personal research and joint researches with international and domestic collaborators are keys for this project. For this purpose we organize various international workshops, interdisciplinary international conferences, and tutorial seminars.

【Expected Research Achievements and Scientific Significance】

Mathematical Aspects: In our project we won't restrict methods and study various problems in a synthetic point of view 'analysis on evolving pattern'. As a result the project is expected to contribute not only to the theory of nonlinear PDEs but also to development of the theory of viscosity solutions, variational analysis, asymptotic analysis, real analysis and stochastic analysis.

Applied Aspects: Problems on evolving pattern spread out in various fields including the theory of crystal growth. If mathematical foundation is strengthened, it is expected to be a key to develop interdisciplinary study.

Numerical example of colliding skrew dislocations on a crystal surface
(Y. Giga, T. Ohtsuka, Y.-H. R. Tsai)

Surface of crystal phlogopite
I. Sunagawa — P. Bennema (1982)

【Publications Relevant to the Project】

Y. Giga, Surface Evolution Equations—A Level Set Approach, Birkhauser (2006) 273pp

Y. Giga and M.-H. Giga, Nonlinear Partial Differential Equations – Asymptotic Behavior of Solutions and Self-Similar Solutions. Kyoritsu (1999) 300pp. (in Japanese)

【Term of Project】 FY2009-2013

【Budget Allocation】 134,500 Thousand Yen

【Homepage Address and Other Contact Information】
http://www.ms.u-tokyo.ac.jp/teacher/giga.html