【Science and engineering（Mathematical and physical science）】

**Theory of global well-posedness on the nonlinear partial differential equations**

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**【Outline of survey】**

We deal with the nonlinear partial differential equations arising from the mathematical physics and biology. In particular, we are interested in the equations which govern the fluid motion. Our goal is to establish a unified theory on well-posedness of the equation such as existence, uniqueness and stability of solution. Concerning the stationary problem, we first investigate the one in the whole space, and then interior and exterior boundary value problems will be discussed In the interior problem, we bring a focus into the effect of topological type of the domain such as numbers genus onto solvability of the equations. In the exterior problems, the behavior of solutions in the suitable norm with the anisotropic weights is closely related to the shape of the obstacle, and the asymptotic decay property at the space infinity will be discussed intensively. As for the non-stationary problem, we make it clear smallness of the given data in the class of scaling invariant norms which guarantee global existence of classical solutions. As a conclusion, we will construct a unified theory on global well-posedness on nonlinear PDE. Here it should be emphasized that global solvability of classical solutions for large initial data to the 3D Navier-Stokes equations are proposed as one of the seven Millennium problems by the Clay institute with each one million dollars' prize. This is one of the themes in this research project.

**【Expected results】**

By charactering various boundary conditions on the harmonic vector fields, we first establish the Helmholtz-Weyl decomposition in L^p-spaces, and then deal with the steady problem for the Navier-Stokes equations under the inhomogeneous boundary data. It is expected that the relation between solvability of the equation and he flux condition on the boundary data will be clarified. Based on the end-point Strichartz's estimate, the L^p-L^q-estimates for the linealized evolution operators have been so fully developed in recent years that we might make further progress in the local and global well-posedness on the nonlinear dispersive and wave equations.

**【References by the principal investigator】**

・Kozono,H, Navier-Stokes equations, Sugaku 54 (2002), 178--202.
・Kozono,H, Ogawa,T., Misawa,M., Perspective in Nonlinear PDE Nippon-Hyoronsha 2007.

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| **【Term of project】 FY2008－2012** | **【Budget allocation】** **136,800,000 yen** (direct cost) |

**【Homepage address】** http://www.math.tohoku.ac.jp/researchfields/kozono.html