New Developments and Interaction between Algebraic Geometry and Integrable Systems

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【Outline of survey】

From the second half of the 20th century to the present, in the research fields of algebraic geometry and integrable systems, there have been great developments and enhancement of theories, and interesting mathematical examples have been discovered. Like the relation between the correlation functions of the quantum cohomology theory and integrable systems, or mirror symmetry conjecture for Calabi-Yau manifolds, many interesting results have been reported, which suggest deep relationship between these two research fields. In this research project, based on the recent progress of two research fields, we aim at establishing the basic mathematical theory for understanding these deep relationships. Moreover, we are expecting to unite these fields and develop a new research field. Concretely, we will construct global moduli spaces of linear connections with singularities and their compactifications as families of algebraic varieties by means of geometric invariant theory. We will analyze Riemann-Hilbert correspondences in details and establish the geometry of monodromy-Stokes preserving deformations. With the aid of this basic theory, we expect to understand Painleve equations and their generalizations from the view points of deformation theory of algebraic varieties, minimal model theory and birational transformations.

【Expected results】

Establishing the geometry of monodromy-Stokes preserving deformations, it becomes possible to understand various aspects of the associated differential equation, such as Painleve property, holomorphic symplectic structures, Lie theoretic construction and birational structures of their phase spaces. Moreover, it enables us to analyze the flat structure or Frobenius structure in details, and as a result we expect to obtain the mathematical understanding of the correlation functions in the quantum cohomology theory, mirror symmetry and string duality.

【References by the principal investigator】

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【Term of project】 FY2007－2011	【Budget allocation】 19,500,000 yen (2007 direct cost)

【Homepage address】 http://www2.kobe-u.ac.jp/~mhsaito/ftop.html