Field:
Mathematics/Informatics/Engineering

Session Topic:
Fluid-Structure-Interaction in Math and Engineering

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1. Introduction

In computer graphics, visual simulation of natural phenomena has become one of the most important research topics. Examples of such phenomena include water, fire, smoke, clouds, and so on. Many methods for simulating these phenomena utilize techniques developed in computational fluid dynamics. Various natural phenomena can be simulated by solving Navier-Stokes (NS) equations. This paper first describes the NS equations briefly. Then, applications of the NS equations to simulations of smoke, clouds, and aerodynamic sound, are explained.

2. Navier-Stokes Equations

The motion of fluids can be calculated by the numerical analysis of the Navier-Stokes (NS) equations. In computer graphics, the incompressible NS equations are often used:

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla)\mathbf{u} - \frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}, \qquad (1)$$

$$\nabla \cdot \mathbf{u} = \mathbf{0}, \qquad (2)$$

where $\mathbf{u}$ is the velocity, t is the time, ρ is a fluid density, p is a pressure, ν is a kinematic viscosity and $\mathbf{f}$ is an external force. In the NS equations, the velocity $\mathbf{u}$ and the pressure p are unknown variables and computed numerically. These equations are often solved by subdividing the simulation space into a number of grid points and by updating the unknown variables according to the discretized version of the NS equations.

3. Simulation of Smoke

Smoke is one of the most straightforward applications of the NS equations. Together with the NS equations, an equation representing advection and diffusion of the smoke density is calculated. A vorticity confinement term is often introduced as an external force in order to model the small scale rolling features of smoke that are absent on most coarse grid simulations [1].

4. Cloud Formation

Cloud formation can be simulated by numerical analysis of atmospheric fluid dynamics. Although there are various types of clouds, the formation process of cumuliform cloud formation is described here. In this case, we need to take into account three quantities (temperature, water vapor, and water droplets) in addition to the velocity and the pressure. The physical process for the cumuliform cloud formation is as follows. First, the air is heated by the ground and this creates ascending air currents. The temperature of the rising air currents decreases due to adiabatic cooling, so vapor in the air parcel causes a phase transition, coagulates, and water droplets are generated. The water droplets are perceived as the cloud. At that time when the phase transition occurs, the latent heat is liberated, which creates additional buoyancy forces and promotes further growth of the clouds. These processes can be expressed by five partial differential equations [2]. Two of them are the NS equations. The other three correspond to temperature, water vapor, and clouds.

4. Aerodynamic Sound

The Navier-Stokes equations are useful not only for visual simulation but also for sound synthesis. There are some types of sounds that are generated by the motion of fluids, such as wind and water. Sound generated by air is called aerodynamic sound and is often generated when stick-like objects are present in a flow. The cause of aerodynamic sound is not the subtle oscillation of a solid object but vortices generated in air. This type of sound can be calculated accurately if compressible NS equations are solved to analyze the pressure fluctuations of air. However, this computation is computationally very expensive. Fortunately, there is a fact that the sound can be predicted by the pressure fluctuations on object surfaces. The pressures on object surface can be calculated by solving the incompressible NS equations. The pressure fluctuations are then converted into the sound. Based on this fact, we developed a real-time method for synthesizing the sound by precomputing the pressure profile [3].

5. Conclusion

We have shown that the Navier-Stokes equations are powerful in simulating natural phenomena. The equations can be used for both visual and sound simulations. There are a number of phenomena that can be simulated by the numerical analysis of the Navier-Stokes equations. This paper shows only a few of them. We hope that our research will stimulate the future researches in this direction.

References

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