Many applications in science and engineering lead to shape identification problems. For example, in inverse obstacle scattering one aims at the determination of the unknown shape of an obstacle from the measurements of the far field of scattered acoustic waves. In particular, this situation is met in radar or sonar. Another important application is the electrical impedance tomography. It is used in medical imaging to reconstruct inclusions of different electric conductivity from measurements of currents and voltages at the surface. The same technique is also used in geophysical explorations or in nondestructive testing.

Shape identification problems are in general formulated as the minimization, with respect to the unknown design parameters to be identified, of a suitable objective function that quantifies the distance (misfit) between experimental data and corresponding model predictions. Shape identification problems are inverse problems and thus in general ill-posed in the sense of Hadamard (1923).

A problem is called well-posed if:

1. There exists a solution to the problem (existence).
2. The problem admits at most one solution (uniqueness).
3. The solution depends continuously on the data (stability).

If one of these properties fails to hold, the problem is called ill-posed. From the mathematical point of view, the existence or uniqueness of a solution can be enforced by enlarging or shrinking the parameter space. Whereas, the stability is of primary concern, which is motivated by the fact that, when the inverse problem lacks the stability property, small inevitable measurement and computational errors are amplified to unacceptable large errors in the solution.

In this talk we survey on the mathematical formulation of shape identification problems as well as their numerical solution. Especially, we consider

1. the discretization of the shape of the obstacle or inclusion,
2. the numerical solution of the underlying partial differential equation,
3. as well as the efficient iterative solution

for the shape identification problem.

Numerical results will be presented to illustrate the methods.

References:

