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海 外 特 別 研 究 員 最 終 報 告 書

独立行政法人日本学術振興会 理事長　殿

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（氏名は必ず自署すること）

海外特別研究員としての派遣期間を終了しましたので、下記のとおり報告いたします。
なお、下記及び別紙記載の内容については相違ありません。

記

1. 用務地（派遣先国名）用務地：　パリ第一大学　（国名：　フランス　）

2. 研究課題名（和文）※研究課題名は申請時のものと違わないように記載すること。
完全性の観点からの証明論的意味論の研究

3. 派遣期間：平成・令和（令和に丸印）　元年　7 月　31 日　～　令和　3 年　7 月　15 日

4. 受入機関名及び部局名
受入機関名：　パリ第一大学
部局名：　科学技術史・科学技術哲学研究所(IHPST)

5. 所期の目的の遂行状況及び成果…書式任意　**書式任意（A4 判相当 3 ページ以上、英語で記入も可）**
（研究・調査実施状況及びその成果の発表・関係学会への参加状況等）
（注）「6. 研究発表」以降については様式 10－別紙 1～4 に記入の上、併せて提出すること。

所期の目的の遂行状況及び成果
(The state and results of my research for the expected goal)

高橋 優太 (Yuta Takahashi)

1 Summary

My research project belongs to the area of philosophy, in particular philosophy of logic. The main purposes of this research project are

(1) to formulate a proof-theoretic semantics based on linear logic, and

(2) to show that a proof system of linear logic is complete with respect to this proof-theoretic semantics.

For these purposes, I adopted a mixed approach which consists of several theoretical frameworks in philosophy, logic and computer science.

Below I first explain the background of my research project (§2). In particular, I will provide brief explanations for proof-theoretic semantics, the completeness with respect to proof-theoretic semantics, and linear logic. Next, I will describe the results in my research project and mention a work in progress (§3).

2 Background

Proof-theoretic semantics, which was introduced by Dag Prawitz during the 1960s and 70s (see e.g. [6, 7]), is a framework to explain the meanings of logical connectives such as "if A then B", "all x are F" and "there is a x being G. The characteristic of proof-theoretic semantics is that it explains the meaning of a logical connective in terms of the inference rules governing this connective, and these inference rules are formulated in the framework of proof theory. Proof theory is an area of mathematical logic which investigates the properties and structures of proofs by treating proofs themselves as mathematical objects (in this sense, proof theory is a metamathematics). For example, a rule governing the logical connective *Conjunction* $\wedge$ is formulated in proof theory as follows: let A and B be two arbitrary propositions, then the *introduction* rule for $\wedge$

$$\frac{A \quad B}{A \wedge B}$$

is an inference rule for the conjunction $A \wedge B$, and this rule says that it is admissible to infer $A \wedge B$ if both A and B are derived. According to proof-theoretic semantics, it is this rule that gives the meaning of the conjunction $A \wedge B$ of A and B; in fact, this rule captures the intended meaning of $A \wedge B$, which states that $A \wedge B$ holds if both A and B hold. Based on this idea, proof-theoretic semantics explains the meanings of more complex propositions such as "For all x being F, there is a y being G such that the relation $R(x, y)$ holds".

So far, the literature of proof-theoretic semantics has proposed several conditions such as *Harmony* in order to explain when a collection of inference rules for a connective is *valid*. Roughly speaking, Harmony requires that there is a balance between the introduction rule for $\wedge$ above and the following *elimination* rules for $\wedge$;

$$\frac{A \wedge B}{A} \qquad \frac{A \wedge B}{B}$$

such a balance can be indeed found between these inference rules. Moreover, one can provide not only the conditions for counting as valid inference rules, but also the conditions for counting as valid *proofs*: given a proof system formulated by means of proof theory, proof-theoretic semantics explains when a proof in this system is a valid one.

Then, by showing the *completeness* with respect to proof-theoretic semantics, one can verify that this semantics captures the proofs in a given proof system exactly. The completeness of a proof system S with respect to proof-theoretic semantics means that any proof in S is valid in the sense of this semantics and vice versa. Hence, if one succeeds in showing that a proof system S is complete with respect to proof-theoretic semantics, then this semantics gives a characterisation of the proofs in S by means of the validity provided by it.

On the other hand, *linear logic* is an area of mathematical logic which was developed mainly in connection to theoretical computer science. Linear logic was introduced by Jean-Yves Girard in the 80s ([2]), and one of its main ideas is the computation as interaction. On the basis of this idea, the research on linear logic has given many insights into the essence of programs including ones of functional programming and logic programming. In proof theory of linear logic, proofs are considered to be programs (as in the so-called Curry-Howard correspondence), and the computation as interaction appears as the transformation of proofs.

The main idea behind my research project is to formulate a proof-theoretic semantics based on linear logic and explain the validity of proofs by using this semantics, which provides an account of the validity in terms of interaction or dialogue. This approach stands in contrast with the approach by traditional proof-theoretic semantics because it is the notion of monologue by an idealised mathematician that underlies the traditional approach.

Then, if one succeeds in showing that a proof system S is complete with respect to my proof-theoretic semantics, a characterisation of the proofs in S in terms of interaction is obtained. This is of conceptual interest, since the characterisation here has an interactive feature which is distinct from traditional proof-theoretic semantics. Moreover, it was shown recently by Thomas Piecha and Peter Schroeder-Heister ([5]) that the completeness with respect to a standard proof-theoretic semantics cannot hold for the proof system of intuitionistic logic, which is one of the main proof systems in mathematical logic. This motivates the research on proof-theoretic semantics to formulate a new proof-theoretic semantics and develop it so that the completeness holds for the proof system of intuitionistic logic.

3 Results (Joint Work with Alberto Naibo)

In joint work with my host researcher Prof. Alberto Naibo, I formulated a linear-logical variant of proof-theoretic semantics by using *Computational Ludics*, which was introduced by Kazushige Terui in [10]. Computational Ludics is a computability- and complexity-theoretic reformulation of Girard's *Ludics* ([3]), and Ludics is one of the standard frameworks in proof theory of linear logic. In particular, we focused on the notion of Harmony in traditional proof-theoretic semantics, and reformulated this notion in Computational Ludics: first, we decomposed the notion of Harmony into the *inversion principle* and the *recovery principle* by following the preceding approaches in proof-theoretic semantics, and then we reformulated these two principles in the framework of Computational Ludics. We called this Ludics-counterpart of Harmony the *harmony condition*. In traditional proof-theoretic semantics, Harmony is utilised for giving an account of the validity of inference rules and proofs, and the harmony condition plays the same role in our proof-theoretic semantics.

On the basis of this reformulation, we proved that the harmony condition is equivalent to each of the two other conditions which are given in terms of the interactive nature of Computational Ludics. This makes it explicit that the notion of validity in our proof-theoretic semantics is based on interaction or dialogue: these two conditions connect our notion of validity with a fruitful notion of interaction which is found in theoretical computer science. Moreover, one of these two conditions is the *internal completeness* in the Ludics framework, and this has the following two advantages. First, we conjecture that the completeness of the linear fragment **MALLP** of polarised linear logic with respect to our proof-theoretic semantics is easily obtained by restricting the more general completeness result by Basaldella and Terui in [1] to our case.[1] The equivalence of the harmony condition to the internal completeness allows to adapt the result in [1] to our proof-theoretic semantics. Second, the internal completeness is a form of completeness which is proper to the Ludics framework: there is no fundamental distinction between proofs and models. Since the completeness with respect to traditional proof-theoretic semantics was an attempt to formulate the notion of completeness of a proof system without model-theoretic means, one can think of our harmony condition as a further development of this attempt.

The extended abstract on the work above was accepted for a contributed talk in Linearity & TLLA 2020 (the second edition of the joint workshop of the International Workshop

[1] However, our research is still far from the goal of finding a proof-theoretic semantics in which the completeness of intuitionistic logic holds.

on Linearity and the International Workshop on Trends in Linear Logic and its Applications) June 29–30, 2020, and I gave the online presentation.[2] After this workshop, the extended abstract was revised as a full paper, and the full paper was submitted for the post-proceedings of the workshop. The paper is currently under review.

The joint work above with Prof. Naibo can be described as an investigation of traditional proof-theoretic semantics from the viewpoint of computation, and I made such an investigation also by a work in progress, which is another joint work with him. Specifically, we discussed the connective denoted by $\bullet$, which was introduced by Stephen Read ([8, 9]), and explained the computational meaning of $\bullet$. The inference rules for $\bullet$ satisfy Harmony in traditional proof-theoretic semantics, but one can derive a contradiction from these rules and so $\bullet$ is problematic from the logical point of view.[3] We argued that the inference rules for $\bullet$ can simulate the *fixed point operator* in partial type theory, which was investigated by e.g. Erik Palmgren ([4]). The inference rule for the fixed point operator implies a contradiction as the inference rules of $\bullet$ do, while the literature on partial type theory revealed several computational significance of this operator. By showing that the inference rules for $\bullet$ can simulate the behaviour of the fixed point operator, we explained that $\bullet$ is still computationally meaningful.

I presented this work at EXPRESS-IHPST online workshop: Truth, proof and communication, June 21–22, 2021.[4] Now I am preparing a full paper through correspondence with Prof. Naibo.

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[2]Accessed August 4, 2021, `https://lipn.univ-paris13.fr/LinearityTLLA2020/`. Linearity & TLLA 2020 is one of the affiliated workshops of FSCD 2020 (the fifth International Conference on Formal Structures for Computation and Deduction) and was originally located on Paris, but it took place as an online workshop due to COVID-19.

[3]On the other hand, the inference rules for the connective $\bullet$ cannot be formulated in the Ludics framework discussed above, so the problem concerning this connective do not emerge in the proof-theoretic semantics by means of Computational Ludics.

[4]Accessed August 4, 2021, `https://inferentialexpressivism.com/events/1041-2/`.

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