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（氏名は必ず自署すること）

海外特別研究員としての派遣期間を終了しましたので、下記のとおり報告いたします。
なお、下記及び別紙記載の内容については相違ありません。

記

1. 用務地（派遣先国名）用務地：Weizmann institute of science　　（国名：　イスラエル　　）

2. 研究課題名（和文）※研究課題名は申請時のものと違わないように記載すること。
トポロジカルチャージの理論的研究

3. 派遣期間：2018　年　4　月　3　日　～　2020　年　4　月　2　日

4. 受入機関名及び部局名
Weizmann institute of science, department of condensed matter physics

5. 所期の目的の遂行状況及び成果…書式任意　**書式任意（A4 判相当 3 ページ以上、英語で記入も可）**
（研究・調査実施状況及びその成果の発表・関係学会への参加状況等）
（注）「6. 研究発表」以降については様式 10－別紙 1～4 に記入の上、併せて提出すること。

Annual report

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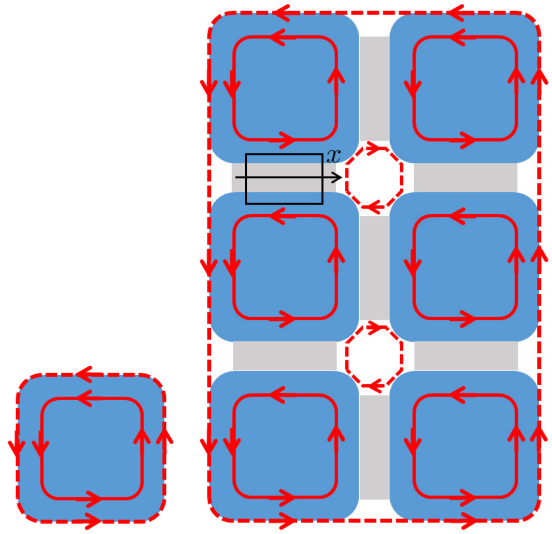
This is the annual report of my research which is based on the joint work with Rohit Kalloor, Alexei M. Tsvelik, Yuval Oreg (to appear)

1 Introduction

One of the pivotal features of topological phases of matter is that they have exotics fractional excitations, the so-called anyons[4]. Generally anyons fall into two categories; Abelian anyons and non-Abelian anyons. Particularly, non-Abelian anyons are of importance as they show non-Abelian statistics, meaning braiding two anyons is characterized by a matrix in a degenerate Hilbert state, which can potentially be used for quantum information process. To be more specific, world lines of anyons can be interpreted as strands in the $2+1$ dimension, and braid group representation of these strands may be associated with unitary operation of a quantum state. A nice thing about this interpretation is that such an unitary operation is topological, hence immune to a local perturbation. This can be intuitively understood by noting that a small deformation of the strands doesn't affect the global configuration of the strands such as the linking number of the strands, over-crossing and under-crossing, e.t.c. Therefore one expects quantum computations realized by braiding anyons are topologically robust.

A prominent example of a non-Abelian topological phase is the Moore-Read state in $\nu = 5/2$ fractional quantum Hall fluid. In this state, the Ising anyon may occur as an excitation. However, in quantum information perspective, the Ising anyon is not universal; braid group representation of the Ising anyons is not sufficient to give an arbitrary unitary operators. It is therefore desirable to obtain a non-Abelian topological phase beyond the Moore-Read state.

In this project, we propose a way to construct topological phases in the networks of interacting integer quantum Hall (IQH) islands. Our phases include the Kalmeyer-Laughlin (KL) state[3], one of the spin liquid phases which has deconfined spin excitation, and a phase which has a special anyon, the so-called Fibonacci anyon which is universal in the quantum information point of view. The basic idea behind our proposal depends upon the network construction, originally suggested by Chalker and Coddington[1] and recently updated by Hu and Kane[2] in the context of interacting p-wave superconductors.



(a) (b)

Figure 1: (a) An integer quantum Hall (IQH) island with filling fraction $\nu = 2k$ which has $2k$ chiral edge modes moving in the counterclockwise direction. These modes are decomposed into the $U(1) \oplus SU(k)_2$ sectors marked by the bold red line and the $SU(2)_k$ sector depicted by the dashed red line. (b)Networks of the IQH islands. The $U(1) \oplus SU(k)_2$ sectors (bold red line) are confined in each IQH island, whereas the $SU(2)_k$ sector (dashed red line) propagates inside the vacuum area or along the entire edge of the system, yielding the $SU(2)_k$ topological phase.

2 Summary of results

2.1 Simplest case – construction of the KL state

Here, we give a simplest case of our construction, that is, the KL state. We prepare an IQH island with filling fraction $\nu = 2$ in a square shape as illustrated in Fig. 1(a). There are two chiral edge modes propagating along the island characterized by two copies of $U(1)$ chiral fields, thus, the chiral edge modes have the $U(2)$ symmetry. A key observation is that $U(2)$ is decomposed as $U(2) = U(1) + SU(2)$, reminiscent of the decomposition into charge and neutral modes in a fractional quantum Hall state. This decomposition is also interpreted as spin-charge separation in the physics of Tomonaga-Luttinger liquid.

A more precise form of this decomposition is described by conformal embedding, which reads as

$$U(2)_1 = U(1) \oplus SU(2)_1 \tag{1}$$

where the number in the subscript represents a level of a current algebra. We don't go into the details of the level any further but still subsequent physical argument can be understood. It turns out that this conformal embedding is suggestive of the KL state, as on the right hand side (r.h.s) of Eq. (1), the $SU(2)_1$ sector appears, which is exactly what characterizes the KL state.

To proceed further we introduce an interaction between adjacent IQH islands. To clarify the mechanism, we suggest the reader to focus on the area inside the back frame as shown in Fig. 1(b). In this area there are two pairs of counter-propagating modes. Denoting $\psi_{R,\alpha}$ ($\psi_{L,\alpha}$) as the Dirac field with $\alpha = 1, 2$ corresponding to edge modes of the top (bottom) IQH island inside the black frame, we define the following currents

$$\begin{aligned}
J^x_{R/L} &= \frac{1}{2}(\psi^\dagger_{R/L,1}\psi_{R/L,2} + \psi^\dagger_{R/L,2}\psi_{R/L,1}), \\
J^y_{R/L} &= \frac{1}{2}(-i\psi^\dagger_{R/L,1}\psi_{R/L,2} + i\psi^\dagger_{R/L,2}\psi_{R/L,1}), \\
J^z_{R/L} &= \frac{1}{2}(\psi^\dagger_{R/L,1}\psi_{R/L,1} - \psi^\dagger_{R/L,2}\psi_{R/L,2}).
\end{aligned} \tag{2}$$

These currents have $SU(2)$ symmetry, allowing us to write them in a more compact form as

$$J^a_{R/L} = \sum_{\alpha,\beta=1,2} \psi^\dagger_{R/L,\alpha}\frac{\sigma^a_{\alpha\beta}}{2}\psi_{R/L,\beta} \; (a = x, y, z) \tag{3}$$

with $\sigma^a_{\alpha,\beta}$ being the $SU(2)$ generators. The Hamiltonian inside the black frame is given by

$$\begin{aligned}
H_2 = \quad & \int dx \sum_\alpha v(i\psi^\dagger_{R,\alpha}\partial_x\psi_{R,\alpha} - i\psi^\dagger_{L,\alpha}\partial_x\psi_{L,\alpha}) \\
& + \sum_{a=x,y,z} \lambda_2 J^a_R J^a_L,
\end{aligned} \tag{4}$$

where v is the velocity of the Dirac fields, x is the one-dimensional coordinate in the frame, and λ_2 is the coupling constant. Our network has the identical interactions Eq. (4) between all adjacent islands, as indicated by areas with grey color in Fig. 1(b).

At $\lambda_2 > 0$ the current-current interaction (*i.e.,* the term in the second line) in Eq. (4) is marginally relevant and gaps out the $SU(2)_1$ sector in Eq. (1), yielding the desired KL state, *i.e.*, the $SU(2)_1$ topological phase. Indeed, the edge mode of the ungapped sector on each island [the $U(1)$ sector in Eq. (1)] tunnels through the interacting area, but the $SU(2)_1$ mode bounces off. As a consequence, the $U(1)$ sectors remain confined to each IQH island [see red bold lines in Fig. 1(b)]. On the other hand, the edge modes of the gapped $SU(2)_1$ sector are not transmitted through the interaction areas and hence become confined to the edge of the vacuum regions. However, as is clear from Fig. 1(b), one chiral mode is free to propagate along the entire edge of the system [red dashed line in Fig. 1(b)], which results in the desired KL state. The suggested mechanism imposes restrictions on the size of the islands: they must be larger than the correlation length $\xi \sim \exp(2\pi v/\lambda_2)$ to allow the gap to develop.

There is only one nontrivial anyonic excitation s in this phase and fusion rule is given by $s \times s = I$. The s anyon is nothing but the semion described above. Physically, such excitation can bind to a h/e vortex in an IQH island, as may occur in a layer structured IQH island.

2.2 Other phases

We can construct other topological phases by simply replacing the $\nu = 2$ IQH islands with the ones at the filling fraction $\nu = 2k$ in the geometry shown in Fig. 1(b). Instead of Eq. (1), utilizing the following conformal embedding

$$U(2k)_1 = U(1) \oplus SU(2)_k \oplus SU(k)_2, \tag{5}$$

and introducing an interaction between adjacent IQH islands in the similar form as the one in Eq. (4), we obtain the $SU(2)_k$ topological phase.

When $k = 2$, we get the $SU(2)_2$ topological phase in the networks of $\nu = 4$ IQH islands. In this phase, there are three types of excitations, I, ψ, σ with fusion rules $\psi \times \psi = I$, $\psi \times \sigma = \sigma$, $\sigma \times \sigma = I + \psi$. This phase behaves as the anti-Pfaffian state, one of a candidate state of a fractional quantum Hall state at $\nu = 5/2$. However, there are a few physical differences in these two phases; the filling fraction of the anti-Pfaffian is $\nu = 5/2$, on the other hand, the $SU(2)_2$ topological phase is constructed by $\nu = 4$ IQH islands. Also, in the anti-Pfaffian state, there are charge modes which propagates along the edge whereas in the $SU(2)_2$ phase, there are only the neutral modes.

The case of $k = 3$ is also of our interest as the obtained phase is the $SU(2)_3$ topological phase which has the Fibonacci anyon – a "holy grail" of the quantum computation.

3 Conclusion

In this project, we have demonstrated that there are rich topological phases in interacting IQH islands which are characterized by the $SU(2)_k$ sector. Especially we focused on the case of $k = 1$, 2, and 3.

Experimental realization of our topological phases is an important issue. In the case of $k = 1$, the current-current interaction in Eq. (4) can be understood in the bosonization language as a combination of backward scattering and density-density interaction. One possible way to tune such an interaction would be controlling the density by adjusting the gate voltage of the quantum Hall sample. Furthermore, if we introduce networks of double-layer IQH islands, the backward scattering between adjacent islands may occur due to momentum and spin conservation. Since the edge mode of the KL state is neutral, the fist step towards confirmation of this state would be measuring thermal conductance.

We have mentioned that in the $SU(2)_3$ topological phase, the Fibonacci anyon may occur as an excitation. However, in this phase, there are other two types of non-trivial anyons which are not universal. Finding a way to excite only the Fibonacci anyons in this phase and furthermore carrying out braiding these anyons is an important remaining issue in the perspective of quantum computation. One of the strategies would be to construct the so-called $(G_2)_1$ topological phase which has only one type of non-trivial anyonic excitation which is the Fibonacci anyon.

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