

Form B-2
(FY2022)
Must be typed

Date (日付)

26/12/2022 (Date/Month/Year: 日/月/年)

Activity Report -Science Dialogue Program-
(サイエンス・ダイアログ事業 実施報告書)

- Fellow's name (講師氏名): Maria Stella Adamo (ID No. P21312)

- Name and title of the lecture assistant (講義補助者の職・氏名)

- Participating school (学校名): Toshimagaoka Girls' High School

- Date (実施日時): 17/12/2022 (Date/Month/Year: 日/月/年)

- Lecture title (講義題目):

All the shades of matrices: from counting to quantum theory

- Lecture format (講義形式):

◆ Onsite ・ Online (Please choose one.) (対面 ・ オンライン (どちらか選択ください。)

◆ Lecture time (講義時間) 60 min (分), Q&A time (質疑応答時間) 30 min (分)

◆ Lecture style (ex.: used projector, conducted experiments)

(講義方法 (例: プロジェクター使用による講義、実験・実習の有無など))

I used slides with math quiz and drawings. I showed pictures at the beginning

- Lecture summary (講義概要): Please summarize your lecture within 200-500 words.

See the file attached.

◆ Other noteworthy information (その他特筆すべき事項):

- Impressions and comments from the lecture assistant (講義補助者の方から、本事業に対する意見・感想等がありましたら、お願いいたします。):

Lecture summary (200-500 words)

In the beginning, I showed some pictures of the places where I carried out my research in Italy. My lecture was divided into four parts. The first part was introductory to matrices and their properties. I defined what matrices are and the operations that can be performed with two or more matrices, discussing the product of square matrices and their non-commutativity. Then I highlighted the differences in the product between real numbers, and I quickly sketched the product of rectangular matrices, motivating why one prefers to look at square matrices.

In the second part, I explained ways of reinterpreting matrices since real numbers can be considered matrices with a single row and a single column. Then I discussed the concept of isomorphism and families of matrices. In particular, I considered one parametrized by a real number that has a group structure and encodes translations. Examples of groups come from geometry, so I analyzed the group of symmetries of a regular triangle. Then I discussed the groups found in arithmetics and how one can think of the clock as a finite group.

The third part goes into the notion of isomorphism. It clarifies why the group of translation matrices is isomorphic as groups to the real numbers with the addition.

Suppose that matrices can be considered as higher dimensional objects resembling real numbers. In that case, we ask if there is a higher dimensional version of real numbers, namely complex numbers, which are also motivated by solutions of algebraic equations. Complex numbers are isomorphic to some class of square matrices. The new point of view allows one to interpret complex numbers and their geometric rotations on the plane. However, in this case, matrices that encode rotations for a fixed radius, for example, for the unitary radius, cannot be isomorphic to real numbers since angles that differ by 360 degrees give rise to the same matrix.

All the matrices considered are finite, so one tries to consider matrices with infinite rows and columns. However, computing the product of two infinite matrices may lead to divergent series. If instead of infinite matrices, one tries to consider vectors with infinite components and tries to rewrite them as a sum, then a similar problem appears. This suggests that one needs to consider limits when dealing with infinite objects, and it may not be easy to check whether the objects are well-defined. Therefore one considers instead maps between infinite objects that preserve the operations and their families, such as operator algebras. Such structures appear in Algebraic Quantum Field Theory, where my research can be placed. In this case, measurements in quantum field theory are encoded in nets of algebras of operators.

I mentioned names of mathematicians who contributed to these topics, and I proposed quizzes to allow the students to understand better.