

Form B-2  
(FY2022)  
Must be typed

Date (日付)  
18/11/2022 (Date/Month/Year: 日/月/年)

**Activity Report -Science Dialogue Program-**  
(サイエンス・ダイアログ事業 実施報告書)

- Fellow's name (講師氏名): Bat-Od BATTSEREN (ID No. P21737 )

- Name and title of the lecture assistant (講義補助者の職・氏名)

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- Participating school (学校名): Fukui Prefectural Fujishima High School

- Date (実施日時): 16/11/2022 (Date/Month/Year: 日/月/年)

- Lecture title (講義題目):

Group theory and Banach-Tarski paradox

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- Lecture format (講義形式):

◆  Onsite ・  Online (Please choose one.) (  対面 ・  オンライン (どちらか選択ください。)

◆ Lecture time (講義時間) 45 min (分), Q&A time (質疑応答時間) 15 min (分)

◆ Lecture style (ex.: used projector, conducted experiments)

(講義方法 (例: プロジェクター使用による講義、実験・実習の有無など))

via Zoom, beamer presentation, used projector

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- Lecture summary (講義概要): Please summarize your lecture within 200-500 words.

In this talk, I wanted to give a very short introduction to my research interest and present one of the most surprising mathematical results called Banach-Tarski paradox. I began by giving a self-introduction about myself and about my background. Then I stated the main theorem I wanted to present: "It is mathematically possible to divide a unit sphere into five pieces, rotate and move them, and make a sphere out of two parts and another sphere out of the remaining three parts" -- Banach-Tarski 1924. For motivational purposes, I presented a similar tricky phenomenon about chocolate cutting using animation and explained that this trick is not satisfactory to our condition. Then I moved to the proof of the Banach-Tarski paradox and gave a very brief introduction to the Group Theory. The theory was too abstract, so I tried to provide as many interesting examples such as operations on Rubik's cube, rotating a geometric object (square, pentagon, hexagon, etc.), and numerical examples that students encounter during high-school mathematics class. Then, I finally explained how the Free group is constructed in two different ways: one with algebraic words, and the other with a Cayley graph. The latter explanation was simpler to understand because it provides an image. This illustration allowed us to understand the Banach-Tarski paradox for the free group. Then we used a free group action on the unit sphere to understand the original Banach-Tarski paradox.

◆ Other noteworthy information (その他特筆すべき事項):

It was very nice unique experience for me to interact with high-school students and to discuss about mathematics.

- Impressions and comments from the lecture assistant (講義補助者の方から、本事業に対する意見・感想等がありましたら、お願いいたします。):