Title of dissertation			
On Positively Graded Unique Factorization Rings and Unique Factorization Modules			
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Let R be a prime ring that is Noetherian and let Q be its quotient ring. Consider a (fractional) ideal A in Q. Define the left R-ideal$(R:A)_l = \{q \in Q \mid qA \subseteq R\}$, and the right R-ideal$(R:A)_r = \{q \in Q \mid Aq \subseteq R\}$. We define a v-operation: $A_v = (R:(R:A)_r)_l \supseteq A$and if $A = A_v$ then A is called a right v-ideal. Similarly, ${}_vA = (R:(R:A)_l)_r$ and A is called a left v-ideal if $A =_v A$. If ${}_vA = A = A_v$, then A is just called a v-ideal in Q. Further, define left order $O_l(A) = \{q \in Q \mid qA \subseteq A\}$ and right order $O_r(A) = \{q \in Q \mid Aq \subseteq A\}$ of A. In 1991, Abbasi et.al. defined a unique factorization ring (UFR for short) by using v-ideal, that is, a ring R is called a UFR if any prime ideal P with $P = P_v$ or $P =_v P$ is principal, that is, $P = pR = Rp$ for some $p \in P$.

Let $R = \bigoplus_{n \in \mathbb{Z}_0} R_n$ be a positively graded ring which is a sub-ring of the strongly graded ring $S = \bigoplus_{n \in \mathbb{Z}} R_n$, where R_0 is a Noetherian prime ring. In this dissertation, it is demonstrated that R qualifies as a unique factorization ring if and only if R_0 is a $\mathbb{Z}_0$-invariant unique factorization ring, and R_1 is a principal (R_0, R_0) bi-module. We give examples of $\mathbb{Z}_0$-invariant unique factorization rings which are not unique factorization rings.

Let M be a torsion-free module over an integral domain D with its quotient field K. In 2022, Nurwigantara et al. introduced the concept of a completely integrally closed module (CICM for short) for investigating arithmetic module theory. A module M is designated as a CICM if, for every non-zero submodule N of M,$O_K(N) = \{k \in K \mid kN \subseteq N\} = D$. Conversely, Wijayanti et al. introduced the notion of a v-submodule. In this context, a fractional submodule N in KM is termed a v-submodule if it satisfies $N = N_v$, where $N_v = (N^-)^+$. Here, $N^- = \{k \in K \mid kN \subseteq N\}$, and $\mathfrak{n}^+ = \{m \in KM \mid \mathfrak{n}m \subseteq M\}$ for a fractional M-ideal $\mathfrak{n}$ in K. Further, in 2022, Wahyuni et.al. defined a unique factorization module (UFM for short) by a submodule approach. A module M is called a UFM if M is completely integrally closed, every v-submodule of M is principal, and M satisfies the ascending chain condition on v-submodules of M. In this dissertation, we prove that if D is a unique factorization domain and M is a completely integrally closed module with the ascending chain condition on v-submodules, then M is a unique factorization module (UFM) if and only if every prime v-submodule P of M is principal, that is, $P = pM$ for some $p \in D$.

Let $M = \bigoplus_{n \in \mathbb{Z}} M_n$ be a strongly graded module over a strongly graded ring $D = \bigoplus_{n \in \mathbb{Z}} D_n$ and $L = \bigoplus_{n \in \mathbb{Z}_0} M_n$ be a positively graded module over a positively graded domain$R = \bigoplus_{n \in \mathbb{Z}_0} D_n$. In this dissertation, we investigated whether the properties found in UFR can be developed in UFM. Some results that can be obtained include: if M_0 is a UFM over

D_0 and D is a UFD, then M is a UFM over D. Moreover, we provide a necessary and sufficient condition for a positively graded module L to be a UFM over a positively graded R.

This dissertation is organized as follows. In Chapter I, we provide the historical research of this research. In Chapter II, we provide some preliminaries regarding graded rings and graded modules. In Chapter III, we provide some results regarding to UFRs. In Chapter IV, we provide some results regarding to UFMs, particularly related to strongly graded modules and positively graded modules. In Chapter V, we end this dissertation with some results on the generalized Dedekind module and future research plans.

Keywords: positively graded ring, positively graded module, unique factorization ring, unique factorization module, generalized Dedekind module.

Photos

Take picture with Prof. Akira UEDA at his office.

Defense for Ph.D. Examination.