Kakenhi and the Research Environment for Young Scientists

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My research fields have been algebraic geometry and complex manifolds, which are part of pure mathematics.

These areas, algebraic geometry and complex manifold theory, have a proud tradition in Japan, having produced three winners of the Fields Medal. These traditional areas are now being advanced with new research, driven by the development of integrable systems including good differential equations such as Painlevé equations; the superstring theory of mathematical physics, notably mirror symmetry conjectures concerning Calabi-Yau manifolds; and various quantum invariant theories. In my current Grant-in-Aid for Scientific Research (S) project titled “Algebraic Geometry and Integrable Systems—Deepening of Theory and New Developments in Mathematics and Mathematical Physics” (2017–21), the aim is to deepen theory by connecting and merging new studies in the two different fields of algebraic geometry and integrable systems, and by incorporating new ideas from mathematical physics.

Looking back, I had the benefit of a wonderful educational and research environment in university and graduate school. I studied under Prof. Kenji Ueno in undergraduate and graduate school. In the master’s program, I delved deeply into the complex manifold theory and elliptic surface theory of Prof. Kunihiko Kodaira. These became the constant core of my research, providing me with real examples when I studied new theories. I was also given the chance early on to study the newly announced Mori theory and minimal model programs for higher-dimensional algebraic manifolds, among other topics. Then there were joint workshops and study sessions with the mathematical physics group working on conformal field theory and superstring theory, giving me the rare opportunity to hear directly from Prof. Toru Eguchi and Prof. Hiroshi Oguri in his younger days.

For young researchers, even after earning a degree and landing a job, it can be a struggle at first to establish the direction of their own research. In my case, as well, it was difficult to decide a path for original research. This may sound a bit like excuse-making, but my initial hesitation was one factor in lacking the confidence to actively apply for Kakenhi as principal investigator. I was fortunate that around that time, in a Grant-in-Aid for Scientific Research for Priority Areas project with Prof. Ueno as principal investigator, on the theory and application of integrable systems with infinite degrees of freedom (1992–97), many world-class researchers were invited and various workshops were planned. I am grateful for the special consideration given to the research environment for young scientists.

Today, the importance of Kakenhi is far greater than ever, and application for funding under this program is now essential for young researchers. For a senior scientist like myself, I believe we are called on to exercise leadership in obtaining funding with a forward-looking, compelling research plan involving young researchers, and breathing new life into that field.

Around 10 years after obtaining my academic degree, in 1996 I was appointed as professor in my present university. Since then, I have actively applied for Kakenhi, and have been extremely fortunate to have applications adopted almost continuously throughout this time. Applications for Grant-in-Aid for Scientific Research (B) were adopted three times (1997–99, 2000–02, 2004–06), followed by Grant-in-Aid for Scientific Research (S) three times (2007–11, 2012–16, 2017–21).

After my first application for Grant-in-Aid for Scientific Research (S) was adopted, in 2007 I launched a website, providing information including notices of workshops. During the decade from 2007 to 2016, I held international workshops 37 times, including two in France and one each in Belgium and Taiwan. Throughout that time, I enjoyed exchanges with researchers from France, Hungary, India, Taiwan, the US, South Korea, and elsewhere, and joint research advanced as well. Young researchers in related fields were also employed as assistant professors. It gives me no end of joy that they are today active as scientists in various organizations.

In my Grant-in-Aid for Scientific Research (B) project, working together with Profs. Shinobu Hosono and Atsushi Takahashi, we looked at Calabi-Yau manifolds that can be obtained from the self-product of a rational elliptic surface. To this day, I strongly remember how we made proposals concerning the counting of curves of arbitrary genus and the relationship to BCOV holomorphic anomaly equations and automorphic forms, as well as concerning integer BPS invariants consistent with the Gopakumar-Vafa conjecture.

Through Grants-in-Aid for Scientific Research (B) and (S) projects, we attempted an algebraic geometric characterization of differential equations classified more than 100 years earlier as Painlevé equations, aiming to demonstrate that differential equation systems obtained by isomonodromic deformation have generally a good property called the geometric Painlevé property. Judging that it would be necessary to engage the cooperation of Assoc. Prof. Michiaki Inaba and Prof. Katsunori Iwasaki, who at the time were faculty members on the Hakozaki Campus of Kyushu University, I proposed collaborative research and made numerous trips between Kobe and Hakozaki in Fukuoka.

By succeeding in constructing moduli spaces of stable parabolic connections as nonsingular algebraic manifolds, and demonstrating that a Riemann-Hilbert correspondence is a bimeromorphic proper surjective analytic map, it is possible to present a complete proof of the geometric Painlevé property of differential equation systems obtained by isomonodromic deformation. These matters were shown for Painlevé equations of type VI and their extension, Garnier systems, in a joint paper by Inaba, Iwasaki, and Saito published in 2006. In Grant-in-Aid for Scientific Research (S) projects, these results were extended to the cases of regular singularities, arbitrary genus, and arbitrary rank by Inaba, and to the case of general unramified irregular singularities by Saito and Inaba.

We are currently attempting to extend these to the case of general ramified irregular singularities. In addition, now that the structures of moduli spaces of connections and Higgs fields, and the structures of moduli spaces of monodromy/Stokes data, have been investigated in detail, our current research themes include the theory of good coordinates, and verification of the geometric Langlands conjecture. Also of deep interest are the theory of minimal models to higher-dimensional algebraic manifolds, and the curious relationship between Painlevé tau functions and conformal block theory, as there still endless topics to explore.