【Grant-in-Aid for Scientific Research (S)】

Science and Engineering (Mathematical and Physical Sciences)

Title of Project : Algebraic Geometry and Integrable Systems - Deepning of Theory and New Developments in Mathematics and Mathematical Physics -

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Research Project Number : 17H06127 Researcher Number : 80183044
Research Area : Mathematical and physical sciences, Mathematics, Algebra
Keyword : Algebraic Geometry, Integrable Systems, Differential Geometry, Moduli Theory, Mirror Symmetry

【Purpose and Background of the Research】

From our construction of moduli spaces of stable parabolic connections with regular singularities over a nonsingular algebraic curve and detailed analysis of Riemann-Hilbert correspondences, one can give a rigorous proof of the geometric Painlevé property of isomonodromic diff. equations. At this point, one can also deal with the construction of moduli space of connections with irregular singularities and their Stokes Phenomenon. Moreover we are interested in mathematical foundations of mirror symmetry, WKB analysis and topological recursion of Eynard-Orantan. We would like to understand the relation between asymptotic analysis of connections and numerical invariants. From the view point of higher dimensional birational geometry, it is interesting to understand the detailed geometric structure of good model of moduli spaces of connections and Higgs bundles. From these backgrounds, our research objectives can be listed below.

1. Study of the geometry of Riemann-Hilbert correspondences (Irregular singularity)
2. Study of the minimal model theory in the birational geometry and its applications to phase spaces and geometric Langlands conjecture.
3. Study of quantum invariants and their correlation functions and mathematical understanding of mirror symmetry.

【Research Methods】

Project members in Kobe will work together with other related researchers from the various fields like differential equations, integrable systems, birational geometry, differential geometry, moduli theory, mirror symmetry, representation theory, derived category and mathematical physics. According to the research purposes, we encourage each project member to promote each individual research and collaborations with other members. In order to share newly obtained results and new problems to be solved, we will organize workshops and research conferences. Keeping in touch with related overseas researchers, we will update the research project and obtain fruitful collaboration with them. Maintaining the home page of the project, we deliver the research information. Employing young PD researchers, we will promote the researches in next generations.

【Expected Research Achievements and Scientific Significance】

We establish the geometric of Riemann-Hilbert correspondence of connections with irregular singularities. This will promote a unified understanding of the symmetry of the integrable system and the theory of asymptotic expansion. In addition, geometric Langlands conjecture will be solved in some special cases. With these, it is expected that the integral system and algebraic geometry will merge, and we will establish the fundamental theory to understand superstring theory and mirror symmetry.

【Publications Relevant to the Project】

[1] F. Loray, M .-H. Saito, “Lagrangian fibrations in duality on moduli spaces of rank 2 logarithmic connections over the projective line”, IMRN, no. 4, 995–1043, (2015).
[2] M.-a. Inaba, M.-H. Saito, “Moduli of unramified irregular singular parabolic connections on a smooth projective curve”. Kyoto J. Math., 53, no. 2, 433–482, (2013).
[3] New developments in algebraic geometry, integrable systems and mirror symmetry (RIMS, Kyoto, 2008)”, Edited by M.-H. Saito, S. Hosno, K . Yoshioka, ASPM, 59, MSJ, (2010).

【Term of Project】 FY2017-2021
【Budget Allocation】 92,000 Thousand Yen
【Homepage Address and Other Contact Information】
http://www2.kobe-u.ac.jp/~mhsaito/ftop.html