

【Grant-in-Aid for Scientific Research (S)】

Science and Engineering (Mathematical and Physical Sciences)



Title of Project : **Advanced Analysis on Evolving Patterns in Nonlinear Phenomena Driven by Singular Structure**

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Research Project Number : 26220702 Researcher Number : 70144110

Research Area : Mathematical and physical sciences, Mathematics, Mathematical analysis

Keyword : Nonlinear analysis (including variational analysis/nonlinear phenomena)

【Purpose and Background of the Research】

The key to analyze evolving patterns is to understand singular structures. We often observe that there develop several singular structures like a pinching droplet, colliding spirals and a planar surface called a facet. Our goal is to make remarkable progress in analysis of evolving patterns by studying nonlinear partial differential equations (PDEs) describing phenomena.

If singular structures are included in evolving patterns, classical notions of solutions of PDEs are insufficient. To overcome this difficulty, an extended notion of a solution is necessary.

Fortunately, due to development of nonlinear analysis for last three decades, by now there are several successful results for describing motion of evolving objects by “weak solutions” of PDEs.

In this project we further introduce effective notions of weak solutions for various nonlinear PDEs including singular structures. We shall study the well-posedness of the problem to analyze evolving patterns. The topics include a total variation flow and a crystalline mean curvature flow. We shall even study higher-dimensional and higher-order problems. We intend to apply our results to the theory of crystal growth, image processing, fluid dynamics. We shall also clarify relation between several mathematical models. For this purpose we develop up-to-date theory of viscosity solutions, variational analysis, functional analysis, asymptotic analysis and real analysis.

【Research Methods】

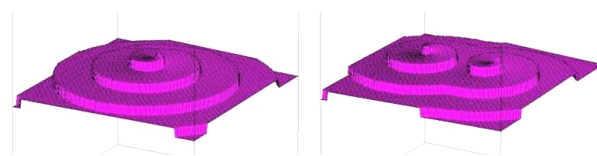
In addition to personal research and joint research with international and domestic collaborators, we will organize international workshops focusing related topics in mathematics, as well as interdisciplinary international conferences and tutorial seminars to find hidden mathematical problems in various disciplines.

【Expected Research Achievements and Scientific Significance】

Mathematical Aspects: We expect to make a breakthrough for longstanding problems and new methods by merging several methods from a

synthetic view point of “analysis on evolving patterns”. We expect new developments on higher-dimensional and higher-order problems. For example, merging variational and viscosity analysis, we expect to extend a level-set method for a crystalline mean curvature flow of a surface to solve an open problem. Developing the theory of viscosity solutions and variational analysis, we expect to contribute to the theory of nonlinear PDEs.

Applied Aspects: Evolving patterns with singular structures appear not only in physics like fluid dynamics or the theory of crystal growth, but also in image processing or differential games in sociology. Fundamental theories of those disciplines are expected to be renovated. For example, analysis on colliding spirals in a crystal surface leads a possibility that the theoretical speed of a growing crystal is precisely calculated. Moreover, fundamental analysis of these equations may lead to applications to another discipline.



Spiral growth by a level-set method
[Ohtsuka, Giga, Tsai (2014)]

【Publications Relevant to the Project】

Y. Giga, Surface evolution equations: A level set approach. Birkhäuser (2006), xii+264 pp.

M.-H. Giga, Y. Giga and J. Saal, Nonlinear partial differential equations: Asymptotic behavior of solutions and self-similar solutions. Birkhäuser (2010), xviii+294 pp.

【Term of Project】 FY2014-2018

【Budget Allocation】 119,800 Thousand Yen

【Homepage Address and Other Contact Information】

<http://www.ms.u-tokyo.ac.jp/~labgiga/>