Title of Project : Innovation in statistics and related mathematics through computational algebraic statistics

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Research Area : Informatics

Keyword : Statistical inference, Computational algebraic statistics

【Purpose and Background of the Research】

The purpose of this project is to bring new innovation to statistics and related fields of mathematics, such as the commutative algebra and the D-module theory, through further developments in computational algebraic statistics.

Takemura, Hibi and Takayama, have been very successfully collaborating on problems of computational algebraic statistics and younger researchers have been joining our collaboration. Close collaboration among statisticians and algebraists of our group is unique even in the world and we aim to make major contribution to the development of computational algebraic statistics on the international level.

【Research Methods】

Takemura, Hibi and Takayama will collaborate on computational algebraic statistics from the background of their own special fields. This collaboration will lead to developments relevant to both statistics and mathematics. More concretely, we further develop toric ideals associated with discrete exponential family of distributions and the holonomic gradient method proposed by us recently. Takemura will pose problems relevant to statistical applications. Hibi will solve them based on his expertise on toric ideals and Takayama will solve them based on his expertise on the ring of differential operators. Takayama will implement the solutions in algebraic computational software.

For developments of holonomic gradient method, we need machinery from various fields, such as the distribution theory of statistics and Gröbner basis for ring of differential operators. This will contribute to innovation in both statistics and related fields of mathematics.

【Expected Research Achievements and Scientific Significance】

Since the breakthrough paper by Diaconis and Sturmfels in 1998, which established the equivalence of toric ideals and Markov bases, the field of computational algebraic statistics developed rapidly and brought relevant results both to statistics and algebra. New important results on Markov bases are still being found and we will contribute further to this area.

The holonomic gradient method was recently proposed in the collaboration of Takemura, Hibi and Takayama. The usefulness of this method is based on the fact that the common probability distributions used in statistics are holonomic functions of both the observation and the parameter. From the general theory of holonomic functions, the normalizing constants and the probability of regions are holonomic in the parameter. Hence these quantities and the maximum likelihood estimation can be evaluated by solving partial differential equations instead of numerical integrations. This will bring reformulation of the whole distribution theory in statistics.

【Publications Relevant to the Project】

- Satoshi Aoki, Hisayuki Hara and Akimichi Takemura. *Markov Bases in Algebraic Statistics*. Springer Series in Statistics, Vol. 199. Springer. 2012.
- Holonomic gradient method for the distribution function of the largest root of a Wishart matrix. *Journal of Multivariate Analysis*, 117, 296-312. Hiroki Hashiguchi, Yasuhide Numata, Nobuki Takayama and Akimichi Takemura. 2013.
- Graver basis for an undirected graph and its application to testing the beta model of random graphs. *Annals of the Institute of Statistical Mathematics*, 65, 191-212. Mitsunori Ogawa, Hisayuki Hara and Akimichi Takemura. 2013.

【Term of Project】 FY2013-2017

【Budget Allocation】 108,600 Thousand Yen

【Homepage Address and Other Contact Information】

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