【Grant-in-Aid for Scientific Research(S)】

Science and Engineering (Mathematical and physical sciences)

Title of Project : Construction of mathematical theory to investigate the macro structure and the meso structure of the fluid motion

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Research Area : Fundamental mathematical analysis
Keyword : Functional equation, mathematical fluid dynamics, stochastic analysis, global mathematical analysis, numerical analysis

【Purpose and Background of the Research】

The mathematical analysis of fluid flow appearing in engineering, such as meteorology, oceanography, medical science, and bioscience is an important issue in modern science. In our research, we focus on the stability of flow of water and air around a moving rigid body and the analysis of multiphase flow concerning the cavitation problem. The rigorous mathematical analysis of these problems can contribute to develop not only mathematical fluid dynamics but also modern science and technology based on fluid engineering.

However, the multi-scale structure of the above problems makes it difficult to analyze them rigorously. The aim of our research is to construct a mathematical theory to understand the both macro and meso structures of fluid motion. To this end, we introduce the following theoretical tools: the $\mathcal{R}$-boundedness, Fourier restriction methods, pseudo-differential operators and finite element methods for the macroscopic level. Furthermore, by jointing with experts on the stochastic analysis, global geometry and nonlinear partial differential equations, we intend to derive equations of motion of viscous fluid with random terms from the mesoscopic motion of fluid molecules, which would provide us a novel knowledge for the mathematical fluid dynamics.

【Research Methods】

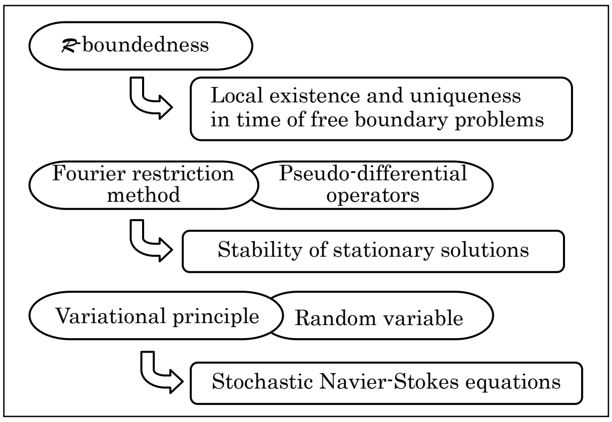

Fig.1: Our research methods

【Expected Research Achievements and Scientific Significance】

From the fact that $\mathcal{R}$-boundedness of resolvents yields the generation of analytic semigroups and the maximal regularity, we can obtain not only the local existence in time of the solution of free boundary problems but also a new way of research general non-linear evolution equations of parabolic type. It is the first attempt in mathematical fluid dynamics to use the Fourier restriction method and pseudo-differential operators for the spectral analysis of the Stokes operator, which would develop the theory of the stability of fluid flow. The elucidation of the structure of the cavitation problem, which is hard to analyze by the macroscopic Navier-Stokes equations, can lead to a unified mathematical theory incorporating the macro and meso structures.

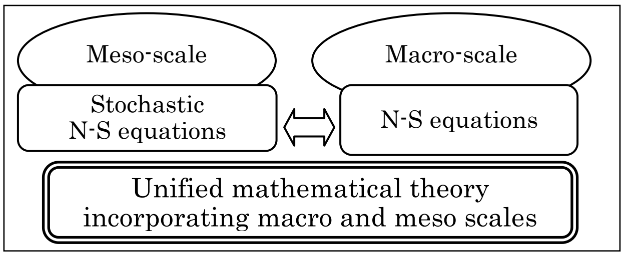

Fig.2: Expected achievements

【Publications Relevant to the Project】

・Y. Shibata and S. Shimizu, On the L_p-L_q maximal regularity of two phase Stokes equations: Model problems, J. Diff. Eqns. **251** (2011), 373-419.
・T. Hishida and Y. Shibata, L_p-L_q estimate of the Stokes operator and the Navier-Stokes flows in the exterior of a rotating obstacle, Arch. Rational Mech. Anal. 193 (2009), 339-421.

【Term of Project】 FY2012-2016
【Budget Allocation】 66,500 Thousand Yen

【Homepage Address and Other Contact Information】

http://www.fluid.sci.waseda.ac.jp/shibata/index.html