【Grant-in-Aid for Scientific Research(S)】

Science and Engineering (Mathematical and physical sciences)

Title of Project : Mathematical Theory of turbulence by the method of modern analysis and computational science

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Research Area : Partial Differential Equations, Nonlinear Analysis
Keyword : Navier-Stokes Equations, Harmonic Analysis, Functional Analysis, Global Well-posedness, Asymptotic Analysis

【Purpose and Background of the Research】

The Navier-Stokes equations have been investigated widely in both theoretical and experimental fields. The mathematical study on the Navier-Stokes equations was founded by Leray .The principal investigator Kozono obtained the optimal decay rate -1 for the energy decay in the 2D case. It turns out that the numerical simulation recently gives the same decay rates. In comparison with the numerical simulation, the advantage of the technique of harmonic analysis makes it possible to handle the asymptotic behavior of physical quantities as the parameter goes to infinity. Such a mathematical method to deal with the exact quantity as the limit of those in finite regions gives us an essential breakthrough in the numerical simulation in theory of turbulence. On the other hand, Kaneda has performed of the computational science and statistical theory of turbulence study. He has the results on the realization of the Direct Numerical Simulation(=DNS) in the world maximum scale about the uniformly isotropic turbulence. Because of such a huge computation, he was awarded the "Super Computing Gordon Bell Prize". In addition, he proceeds to the DNS of the high Reynolds number with the world maximum size about the turbulence between two parallel flat boards. Kaneda is evaluated as the leader developing the spectrum statistics theory of turbulence.

【Research Methods】

This project aims to establish a new theory of nonlinear dynamics for super large degree of freedom including the turbulence in the fluid mechanics in terms of the modern mathematical analysis and the computational science. So, we propose the following four projects (i), (ii), (iii);

(i) Harmonic analytics, singular limit and estimates of effect on the finiteness

(ii) Elucidation of the universal law of turbulence; small and large scales

(iii)Information abridgement technique, prediction possibility and reliable evaluation

The method dealing with infinity and the mathematical analysis such as the limiting procedure give us the elucidation of the turbulence phenomenon requiring a large-scale calculation. We will improve a poor turbulence theory without rigorously mathematical convention, and then construct a new knowledge of turbulence with information abridgement technique. As a result, reliable turbulence theory which do not depend on a law learned by experience or by intuition excessively will be largely accelerated.

【Expected Research Achievements and Scientific Significance】

It is well-known that the Clay Math. Institute proposes seven important Millennium problems, where the existence of the global classic solution to the Navier-Stokes equations is selected. On the other hand, our DNS of the uniformly isotropic turbulence is by far the larger computational performance so that it can deal with the turbulent fluid with the high Reynolds number without any error of the experiment and indeterminacy. Our study is based on the DNS of such a world highest standard of Kaneda's research group, and we are going to overcome difficulty of turbulence with the high Reynolds number. In this way, our research projects develop the modern mathematical analysis, the applied mathematics, computational science and hydrodynamics and will lead the relevant subjects to the world-wide level.

【Publications Relevant to the Project】

・An L^q-approach to Stokes and Navirer-Stokes equations in general domains. Acta Math. 195 (2005), 21--53

・Leray's problem on the stationary Navier-Stokes equations with inhomogeneous boundary data, Math. Z. 262 (2009), 27--39

【Term of Project】 FY2012-2016

【Budget Allocation】 147,000 Thousand Yen

【Homepage Address and Other Contact Information】

http://www.math.sci.waseda.ac.jp/math/