Title of Project : New developments in infinite groups and geometry

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Research Area : Geometry
Keyword : Topology, infinite groups

【Purpose and Background of the Research】

   The symmetry of objects usually appear as infinite noncommutative groups. Compared with finite groups, they have been more difficult to understand. Recently, however, we began understanding better the Lie groups and their discrete subgroups, the diffeomorphism groups preserving a certain structure, the mapping class groups of surfaces, finitely presented groups, groups acting on real trees, etc. On the other hand, theories have been developed on characteristic classes for the group actions, on stable commutator length, on analytic, probabilistic or dynamical approach of infinite groups actions.

   In this project, collaborating with the researchers of related areas, we clarify the relationship between topological, geometric or dynamical properties of group actions and various invariants of them. We aim to find new invariants and their application.

【Research Methods】

   We promote the research on the following 5 items which relate with each other.
(1) [Space forms and infinite groups] We look at the deformation of discrete isometric actions on space forms with indefinite metrics or the action on the spaces at infinity of them.
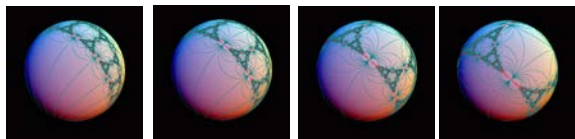

Fig. 1. Deformation of the Apollonian
       gasket appearing as the limit set.

(2) [Group of diffeomorphisms] For the groups of symplectomorphisms, of contactomorphisms, or of diffeomorphisms preserving foliations, we study their topology as well as the topology of their classifying spaces.
(3) [Mapping class groups of surfaces] We clarify the relationship between invariants for mapping class groups and those for diffeomorphism groups. We clarify the relationship among curve complexes, geodesic laminations and fat graphs.
(4) [Dynamical invariants for infinite groups] We study invariants coming from dynamics on invariant subsets, as well as those defined in geometric group theory. We study existence problem, uniqueness problem and conjugacy problem for several group actions.
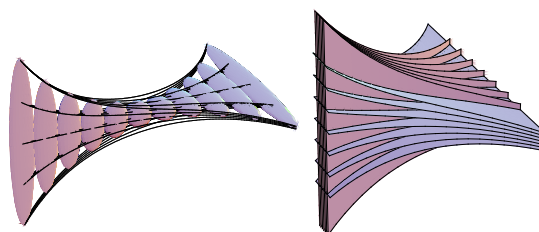

Fig.2. Anosov actions are described by dynamical
   invariants related to the ratio of expansion.

(5) [Infinite groups and wild spaces] We study the actions on the compatifications. We study the spaces appearing as minimal sets for infinite group actions. We also study several compactifications of the Teichmuller spaces.

【Expected Research Achievements and Scientific Significance】

By establishing new method to study infinite noncommutative groups, we understand better important groups eg. diffeomorphism groups. Then we shall obtain a unified view on various geometric objects.

【Publications Relevant to the Project】

Takashi Tsuboi: On the uniform perfectness of the groups of diffeomorphisms of even-dimen- sional manifolds, Commentarii Mathematici Helvetici, 87, (2012) 141-185.
Takashi Tsuboi: On the group of real analytic diffeomorphisms, Annales Scientifiques de l'Ecole Normale Superieure, 49, (2009) 601-651.

【Term of Project】 FY2012-2016
【Budget Allocation】 156,700 Thousand Yen
【Homepage Address and Other Contact Information】
   http:// faculty.ms.u-tokyo.ac.jp/users/IGAG/