

【Grant-in-Aid for Scientific Research(S)】

Science and Engineering (Mathematical and physical sciences)



Title of Project : Proof of Homological Mirror symmetry

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Research Area : mathematics

Keyword : Differential Geometry, Topology, Complex manifold, Algebraic geometry

【Purpose and Background of the Research】

Mirror symmetry is a conjecture proposed in 1990's by physicists.

It becomes famous among mathematicians by the conjecture (due to Candelas etc.) that the generating function obtained by counting the number of rational curves of Calabi-Yau manifolds satisfies certain differential equation obtained from the deformation theory of another Calabi-Yau manifold, that is called its mirror.

This conjecture itself is solved in many cases by Givental etc. and is called the classical mirror symmetry.

Homological Mirror symmetry is proposed by M. Kontsevich in 1994 and deepen the mirror symmetry conjecture.

It conjectures an equivalence of a category obtained from Lagrangian Floer theory of a symplectic manifold (Fukaya category) and the derived category of coherent sheaves of its mirror.

The conjecture now can be stated rigorously when Lagrangian Floer theory is rigorously established in the general setting by Fukaya-Oh-Ohta-Ono.

The purpose of this research is to prove it.

【Research Methods】

Two methods are proposed toward the proof of homological mirror symmetry conjecture.

One is to use the family Floer homology and the other is to prove it first for toric manifolds and generalize it to its submanifolds.

The first one is closely related to Strominger-Yau-Zaslow's proposal to realize a mirror manifold as a dual manifold. The idea to use family of Floer homology to prove homological mirror symmetry is obtained around 1997 by Fukaya-Kontsevich. We are now close to realize it. The second idea is related to the proof by Givental etc. of classical mirror symmetry and also to Hori-Vafa plan of the proof of mirror symmetry.

Fukaya-Oh-Ohta-Ono's recent study of Floer homology of toric manifolds is a way to perform the proof along this line.

【Expected Research Achievements and Scientific Significance】

The proof of homological Mirror symmetry has the following application to symplectic topology.

Theory of pseudoholomorphic curves are very important method in global symplectic geometry. Gromov-Witten invariant Floer homology etc. are important invariant obtained by using it.

One of the main difficulty to study and use symplectic invariant obtained by using pseudoholomorphic curves is its calculation.

At the heart of the construction there is a moduli space of pseudoholomorphic curves. So to calculate those invariant directly from definition requires to determine the set of solution of certain nonlinear PDE that is of course very difficult in general.

(Homological) mirror symmetry provides a way to calculate those invariant by moving the problem to one in complex geometry.

The proof of Homological mirror symmetry is of course useful to the understanding of Mirror symmetry and string theory.

【Publications Relevant to the Project】

K. Fukaya, Y.-G. Oh, H. Ohta and K. Ono, Lagrangian intersection Floer theory-anomaly and obstruction,} International Press/Amer. Math. Soc. (2009).

K. Fukaya, Y.-G. Oh, H. Ohta and K. Ono, Lagrangian Floer theory on compact toric manifolds I, Duke Math. J. 151 (2010), 23-174. ibid II Selecta Math. New Ser. (2011) 17:609-711

Cyclic symmetry and adic convergence in Lagrangian Floer theory, Kyoto J. Math. 50 (2010) 521--590

【Term of Project】 FY2011-2015

【Budget Allocation】 81,600 Thousand Yen

【Homepage Address and Other Contact Information】

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