Title of Project : Study of Geometric Moduli Theories and Theoretical Applications

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Research Area : Mathematics, Algebra
Keyword : Algebraic Geometry

【Purpose and Background of the Research】
Some of geometric objects are described by several parameters uniformly. The space determiend by the parameters is called moduli space. The set (the space) consisting of all the natural and stable limits is called compactifi -cation of the moduli space. There are some applications of compactifications of the moduli spaces. Our progress so far is as follows :
1. Nakamura [1][2] constructed two compactifi -cations of the moduli space of abelian varieties with non-commutative levels and proved that The first compactified moduli consists of all the limit objects whose Hilbert points are stable.
2. In his collaboration with statistical economists Kamiya and Takemura, Terao [6] gave a formula of the number of priority models, identifying the moduli space of priority with the moduli space of the complements of hyperplane arrangements.
3. Ono [3] proved that the Betti number version of the Arnold conjecture concerning the upper bound of the number of fixed points is true if any fixed point of a Hamiltonian mapping of a closed symplectic manifold is nondegenerate.
4. Iwasaki [4] formulated Painleve equations moduli-theoretically, and made an extensive study on entropy and chaotic structure of the dynamical system associated with analytic continuation of solutions of the Painleve eqs.
The purpose of this project is to more deeply study the above compactifications to make theoretical applications to distinct fields.

【Research Methods】
(1) Describe in detail the structure of the compactification of moduli space of abelian varieties at good primes. For it, we describe in detail the natural equations defining (degene -rate) abelian varieties, and construct primary Siegel modular forms as coefficients of the equations. Also extend the compactification of moduli space of AVs to bad primes. For it we generalize Katz-Mazur[5] to higher dimension, using Dieudonne theory,.
(2) Make a deeper study of hypergeometric functions, the complements of hyperplane arrangements and pirimitive integrals through algebraic study of hyperplane arrangements.

Apply arrangements (algebra) to social scienece through collaboration with social scientists.
(3) Study the connection between the mirror symmetry of Langrange-Floer theory (differen -tial geometry) and the mirror symmetry of deformation theories of Potetail functions, or Givental's theory of mirror symmetry (algebraic geometry and physics).
(4) Using the compactification of the moduli (algebraic geometry) we make similar studies on entropy, chaotic structure of the dynamical system for the other types of Painleve equations.

【Expected Research Achievements and Scientific Significance】
In **(1)**,the moduli functor is representable so that any primary Siegel modular forms will appear in principle. Generalization of Katz-Mazur [5] is a long-standing problem. We may be able to exactly describe the structure of the extended compactifications in detail. It is noteworthy of **(2)** that algebra can be applied to social science. The study of mirror symmetry in **(3)** and the algebro-geometric study of Painleve eqs in **(4)** are new and notable internationally.

【Publications Relevant to the Project】
[1] I.Nakamura, Stability of degenerate abelian varieties, Inv. math., vol. 136, 659-715 (1999).
[2] I. Nakamura, Another canonical compactifi -cation of the moduli space of abelian varieties, ASPM of MSJ., vol. 58, pp. 69-135, (2010).
[3] K.Fukaya, K.Ono, Arnold conjecture and Gromov-Witten inv., Topology, vol. 38 (1999).
[4] M.Inaba, K.Iwasaki, M.-H.Saito, Moduli of stable parabolic connections,…,geometry of Painleve equation of type VI, part I, Publ. Res. Inst. Math. Sci., vol. 42, 987-1089 (2006).
[5] N.M.Katz and B.Mazur, Arithmetic moduli of elliptic curves, Princeton (1985).
[6] H. Kamiya, A. Takemura, H. Terao, Periodicity of hyperplane arrangements …, J. Alg. Comb.,vol. 27, 317-330 (2008).

【Term of Project】 FY2011-2015
【Budget Allocation】 139,300 Thousand Yen
【Homepage Address and Other Contact Information】
http://www.math.sci.hokudai.ac.jp/~nakamura/