【Grant-in-Aid for Scientific Research(S)】
Science and Engineering (Mathematical and physical sciences)

Title of Project : Lattices, automorphic forms and moduli spaces

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Research Area : Mathematical and Physical Sciences, Mathematics, Algebra

Keyword : Algebraic Geometry

【Purpose and Background of the Research】

The main problem of algebraic geometry is to study structures and symmetries of algebraic varieties and their moduli spaces. The most interesting example of algebraic varieties is an elliptic curve, whose theory was established in the 19th century, and still is a model of present mathematics, where algebra, geometry and analysis are harmony. The theory of elliptic curves is still interesting, for example, it is applied to the theory of cryptography. A K3 surface is a 2-dimensional analogue of an elliptic curve, which was also found in the 19th century. The name "K3" derives from the initials of three Mathematicians "Kummer, Kähler, Kodaira" and also from the name of the mountain "K2" in the Karakorum. Through the mirror symmetry conjecture, K3 surfaces are interesting to theoretical physics, and there is a mysterious connection between symmetries of K3 surfaces and the Mathieu group, a sporadic finite simple group. Moreover, the theory of periods was established as an analogue of that of elliptic curves. This theory of periods of K3 surfaces matches with the new theory of automorphic forms due to Borcherds.

The purpose of this research is the study of symmetries and moduli of K3 surfaces and Calabi-Yau manifolds from a wide angle, not only by algebraic geometry but also by group theory and automorphic forms.

【Research Methods】

The above K3 surfaces, finite groups and Borcherds' automorphic forms are related via lattices. Our main method is lattice theory. The topological invariant, called Euler number, of K3 surfaces is 24. On the other hand there exist special classes of lattices of rank 24, for example the Leech lattice, which have good properties. We employ these classes of lattices to study K3 surfaces. Also the period domain of K3 surfaces is a bounded symmetric domain of type IV associated to a lattice, on which Borcherds' theory of automorphic forms works. In case of elliptic curves, the classical theory of modular functions and modular forms were very important to study their moduli space. We shall apply Borcherds theory to the case of K3 surfaces.

In mathematical research, it is important to communicate with other researchers in several areas. To do this, we plan to visit other institutes, invite researchers to our institute, and organize international workshops.

【Expected Research Achievements and Scientific Significance】

The classical theory of moduli of curves uses the theory of Jacobians and abelian varieties. A new idea is to use the pairs of K3 surfaces and their automorphisms instead of abelian varieties (although our method works only for curves of small genus). Instead of abelian varieties and theta functions, our application of Borcherds theory to the moduli of K3 surfaces may induce a new method in the theory of moduli. Also our viewpoint between K3 surfaces, finite groups and automorphic forms may become a model for a new theory.

【Publications Relevant to the Project】

・Shigeyuki Kondo, Niemeier lattices, Mathieu groups, and finite groups of symplectic automorphisms of K3 surfaces (Appendix by Shigeru Mukai), Duke Math. J., vol.92, 593—603 (1998).
・Shigeyuki Kondo, A complex hyperbolic structure for the moduli space of curves of genus three, J. reine angew. Math., vol. 525, 219—232 (2000).
・Shigeyuki Kondo, The moduli space of Enriques surfaces and Borcherds products, J. Algebraic Geometry, vol.11, 601—627 (2002).

【Term of Project】 FY2010-2014

【Budget Allocation】 58,600 Thousand Yen

【Homepage Address and Other Contact Information】

http://www.math.nagoya-u.ac.jp/ja/people/faculty-03.html#kondo