## Non-commutative Valuation Rings and Their Global Theories

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Let $R$ be a Dubrovin valuation ring in a simple Artinian ring $Q$. Inspired by [BMU] which they described all right primary ideals of $R$, we investigate the structure of $R$-ideals of $Q$. If $I$ is an $R$-ideal and $I$ is not finitely generated as a right $R$-ideal such that $O_r(I) = \{q \in Q \mid Iq \subseteq I\} = S = O_l(I) = \{q \in Q \mid qI \subseteq I\}$ and suppose that $J(S)$ is Archimedean, it is proved that $I = cA$ for some $c \in$ st$(S)$ and $A$, a right and left $J(S)$-primary ideal. In the case $Q$ is finite dimensional over its center, we obtain: (1) If $I$ is finitely generated as a right $R$-ideal, then $I = cR = Rc$ for some $c \in$ st$(R)$, (2) If $I$ is not finitely generated as a right $R$-ideal such that $J(S)$ is Archimedean, then $I = cA = Ac$ for some $c \in$ st$(S)$ and $A$, a right and left $J(S)$-primary ideal, (3) If $I$ is not finitely generated as a right $R$-ideal such that $J(S)$ is limit prime, then $I$ is one of the following three; $I = cS = Sc$ for some $c \in$ st$(S)$, $I = cJ(R) = J(R)c$ for some $c \in$ st$(R)$ and $I = \cap c_\lambda R_\lambda$ for some $c_\lambda \in$ st$(R_\lambda)$, where $R_\lambda = R_{P_\lambda}$ and $P_\lambda$ runs over all Archimedean prime ideals with $P_\lambda \subset J(S)$.

A ring is called *right (left) bounded* if any essential right (left) ideal contains a non-zero (two-sided) ideal. A ring is just called *bounded* if it is both right bounded and left bounded. Let $S$ be a ring. We say that $S$ is *fully bounded* if $S/P$ is bounded for any prime ideal $P$ of $S$. Let $R$ be a Dubrovin valuation ring in a simple Artinian ring $Q$ and let $P \in$ G-Spec$(R)$, the set of all Goldie prime ideals of $R$, with $P \neq J(R)$ and set $P_1 = \cap\{P_\lambda \mid P_\lambda \in$ G-Spec$(R)$ with $P_\lambda \supset P\}$. Then, in [BMO,(6)], they have shown that the following four cases only occur:

(1). $P$ is lower limit, i.e., $P=P_1$ . Otherwise, $P_1 \supset P$ is a prime segment.

(2). $P_1 \supset P$ is Archimedean.

(3). $P_1 \supset P$ is simple.

(4). $P_1 \supset P$ is exceptional, i.e., there exists a non-Goldie prime ideal $C$ such that $P_1 \supset C \supset P$.

With this classification, it is proved that for a Dubrovin valuation ring $R$ of a simple Artinian ring $Q$, $R$ is fully bounded iff (1) and (2) only hold.

For any regular element $c$ in $J(R)$, we define $P(c) = \cap\{P_\lambda \mid P_\lambda \in$ G-Spec$(R)$ with $c \in P_\lambda\}$, a Goldie prime ideal. $R$ is called *locally invariant* if $cP(c) = P(c)c$ for any regular element $c$ in $J(R)$. Let $R$ be a Dubrovin valuation ring of a simple Artinian ring $Q$. It is shown that $R$ is fully bounded if and only if it is locally invariant.

An Ore domain $S$ is called *right (left) v-Bezout* if $I_v$ is a principal for any finitely generated right ideal $I$ of $S$. $S$ is said to be *v-Bezout* if it is right $v$-Bezout as well as left $v$-Bezout. This ring is a non-commutative version of commutative GCD-domains. Inspired by [Gi], it is proved if $V$ is a total valuation ring of a division ring $K$, then $R = V[x^r, \sigma \mid r \in \mathbb{Q}_0]$ is $v$-Bezout where $\mathbb{Q}_0$ the set of nonnegative rational numbers, $\sigma : \mathbb{Q}_0 \to \mathrm{Aut}(V)$ is defined by $\sigma(r+s) = \sigma(r).\sigma(s)$ for any $r,s \in \mathbb{Q}_0$, and the multiplication in $R$ is defined by $x^r a = \sigma(r)(a)\, x^r$ for any $a \in V$ and $r \in \mathbb{Q}_0$.

In [Mo], he studied PI Prüfer rings under some conditions such as; integral over its center or the center is commutative Prüfer. By using some results in [MMU] and [Mo], we investigate prime ideals of any overring of a PI (Polynomial Identity) Prüfer ring. Let $S$ be an overring of a prime Goldie ring $R$. Suppose that $R$ is Prüfer satisfying a polynomial identity. It is shown that $\mathrm{Spec}(S) = \{PS \mid P \in \mathrm{Spec}(R)$ with $PS \subset S\}$ and $S = \cap R_P$, where $P$ runs over all $P \in \mathrm{Spec}(R)$ with $PS \subset S$.