Geometry and Analysis for Classical Fields

Tohru Ozawa
(Professor, Department of Mathematics, Hokkaido University)

【Outline of survey】

Studies on classical field equations such as nonlinear Schr o dinger and Klein-Gordon equations provide us with a mathematical foundation of Quantum Field Theory as well as an important subject from a mathematical point of view, especially in the framework of mathematical analysis of nonlinear partial equations. Mathematical methods of classical field equations started around sixties with pioneering works by K. J o rgens (Das Angfangswertproblem im Grossen f u r eine Klasse nichtlinearer Welleng eichungen, Math. Z. **77** (1961) 295 — 307) and I. E. Segal (Nonlinear semigroups, Ann. of Math. **78** (1963) 339 — 364). More than forty years have passed since then. Now we have a number of deep theories of specific scalar field equations such as nonlinear Schr o dinger equations, nonlinear wave equations, nonlinear Klein-Gordon equations, Korteweg-de Vries equation, Benjamin-Ono equation, etc. It should be remarked that, according to the progress of mathematical analysis of classical field equations, a great advance has been made in related studies such as theory of function spaces, theory of evolution equations, harmonic analysis, real analysis, etc. Mathematical understanding of classical fields described as systems of equations, however, is still far from the goal made by I. E. Segal and W. A. Strauss in seventies. The objectives of this research project are to clarify mathematical structure of classical fields on the basis of methods built up so far for specific scalar field equations with emphasis on a geometric point of view. Details are omitted.

【Expected results】

By this research project, we expect a better understanding of mathematical structure of nonlinearity in classical fields, a good interaction with harmonic analysis, and a fruitful collaboration with research groups in France, Italy, Mexico, etc

【References by the principal researcher】

(1) S. Machihara, M. Nakamura, K. Nakanishi, and T. Ozawa, Endpoint Strichartz estimates and global solutions for the nonlinear Dirac equation, Journal of Functional Analysis (in press).
(2) S. Machihara, K. Nakanishi, and T. Ozawa, Nonrelativistic limit in the energy space for nonlinear Klein-Gordon equations, Mathematische Annalen **322** (2002), 603 — 621.
(3) M. Nakamura and T. Ozawa, Small data scattering for nonlinear Schr o dinger, wave and Klein-Gordon equations, Annali della Scuola Normale Superiore di Pisa, Serie V, **1** (2002) 435 — 460.

【Term of project】 FY 2004 — 2008 | 【Budget allocation】 66,700,000 yen

【Homepage address】 http://coe.math.sci.hokudai.ac.jp/